The Crown Jewel Of Didactical Gems

A Natural Explanation Of The Present Concept Of Randomness

A Road To A New Definition That Will Alter Mathematics

In my web site cognum.org I created a special section called Didactical Gems.
Subjects as diverse as the Rubic Cube, the game of Nim, Sophie Germain's theorems or the basic syntax of everyday English can be found there. My decision to place an article there was based on two criterias. Self containedness and simplicity.
The usual abstract junk that people can find on the internet, including the Wikipedia math and science articles, are a turn off for any eager person who really want to learn.
But this title is promising much more than simple understanding.
It promises something almost mysterious and indeed it will deliver such thing.
I have to start with some details that don't relate to the content, rather how this article was born.
In a sense, it started in 1965 when I was in math high school in Budapest. But in a stricter sense it started less than a week ago when I woke up from a dream that finally ended a whole earlier week when I had dreams that I wasn't able to even recall. They were abstract and tried to bring back something that I only knew was very important. So then finally I was able to recall the thought that was already captured once.
I got into that high school fifty years ago accidentally. I was not good in math in elementary school at all, or so I believed. There were signs and a substitute teacher who made some remarks contrary to this belief. But the real reason I still applied at all to this high school, was my mother's blind faith in me. To think about what could have happened if I didn't apply, fills me with horror.
Already in the first year I won competitions just few months after that I had to learn even the basic math that my classmates all knew.
The luck of getting in, was not the only factor. On the first math class I received a double of two highest mark because I "solved" a problem that the teacher mentioned to us. He wanted to illustrate how a child can be smarter that a professor by telling that a famous Hungarian mathematician visiting the Russian elementary school Olympiads were asked the following question. We subtract from every letter the x variable and multiply them together that is form $(a – x)\ (b – x)\ \ldots$
What will be the final expression if we calculate it by multiplying these members, that is eliminating the brackets? I wasn't even be able to multiply two members at that time and the question's full meaning was totally beyond my knowledge. What's worse, I didn't even know that a product is zero if any member is zero. In my naïve mind I only knew that something multiplied by zero, definitely becomes zero. So my wishful thinking made me falsely believe for a second that the letter x is the last letter of the Hungarian alphabet. Which is of course not true because y and z come next just like in English. But before my mind could correct my mistake, my hand was up and said that: "Since the last letter is x and $(x – x)$ is zero, thus the result is zero". The teacher ignored my mistake or didn't even notice it because I still got the point that the result is zero, we don't have to calculate anything.
After school I walked out and didn't turn left toward the trams rather went a bit right on Huba street and crossed to the other side to a small church that I noticed earlier in the morning. It seemed neglected but to my surprise the door was a bit opened. I went into the dark small hall and sit down on a bench. That was the first time I entered a church alone. I can not recall what went on my mind but I know that I changed.

The competitions, the will to accomplish, to succeed, to write novels and poems, to start to play the guitar that my older brother wanted to throw out, were the externally visible signs of who I became in such a short time. But there was a hidden world. A secret connection to the other side. So while I was collecting the accepted facts of the world, the known mathematics of the world, the known literature, the known guitar chords, I knew already something much deeper. Namely, that this is just a dead collection of knowable details and that real knowledge comes to you directly.
But this itself was a secret that I never shared with anyone and probably didn't even admit it to myself.
Now when I look back from my knew didactical wisdom and militant view about the poison of society, it again fills me with horror that I could have missed those later big changes and remained in a denial of the other side. So it is a mystery how I could interpret back then those concrete thoughts that simply came from nowhere.
Art as an influence from the other side is easy to alter into simply being talented. With ready scientific visions it is much more puzzling. So, strangely the talented scientists believe more in a concrete other side than the artists. Of course the reflection, the admitted opinions are opposite. The artists blubber, the scientists remain silent.
To be concrete, there were three big mathematical directions that came into my head from nowhere. Non standard analysis, forcing, and randomness.
The new numbers that are filling the points of real space fired me up so much that I wrote letters to Nobel prize winners. Only Heisenberg replied. You could imagine my astonishment few years later when I found out that Robinson already discovered Non Standard Analysis and especially when I found his prologue to draw the attention of physicists to the field.
The foggy concepts of my naïve mind how finite sets can force the behaviour of infinites I was able to tell Cohen the inventor of Forcing himself when I met him later. By that time I had something more important to tell him too.
The third field, randomness was the only one that I systematically started to work out.
I discovered what now is called the Solovay randomness or rather its simplification by Downey. Strangely, recently when I started to deal with randomness again, I contacted Downey before I even learnt that he made the final simplification of Solovay's definition.
Solovay simplified the first accepted definition of randomness made by Martin Löf.
So how could I at the age of sixteen come up with the simplest final form?
Well I will explain this exactly and very simply, so that any first grade high school student will understand it!
This would be enough reason to call this article the crown jewel.
But now I have to get back to the present, that is to less than a week ago when my dream brought back that thought that went actually beyond this definition.
The next day it still sank back to a foggy state. Then when I re-formulated it precisely and saw its revolutionariness, I wrote a four page article called "expectability" and placed it into the New Math section of cognum and wanted to send it to publishers.
This morning just before doing this, I went into some further details of how the reaction could go towards my proposal and this initiated a totally basic approach to first explain the accepted randomness concept and to show along the new idea.
This way my remarks about the Martin Löf randomness will be explained after these.
And this should be so because these are still open questions whereas the existence of the new definition and its amazing consequences are not depending on these.

So here we go! No math knowledge is required, we are only going to flip coins!
To pretend that we are serious, we'll use 0 and 1 instead of the heads and tails.

Probably, you heard about the Law Of Large Numbers that says that in the long runs the heads and tails, sorry, the 0-s and 1-s both will become about half half.
I promised no math, and you should especially forget this law because our whole point will become how false this is.
On the other hand I will call something else the Law Of Big Numbers, that is indeed simple and crystal clear to you already, without opening any books.
This law says that whatever has some non zero chance, must come true if you try it long enough. You might say that this is the cause that people buy lottery tickets.
To win has very little chance but if we lived for ever and tried every week, then sooner or later we would win. The times that such winning could be expectable is obviously the same number of weeks how many combinations are possible. So to express it in weeks or years would be even better to see how hopeless is to play.
But in these sentences we touched on something very deep beyond the stupid reality of gambling. You could say, we only wanted to flip coins, so why am I talking about lottery. So firstly, lets replace the lottery with coin flip. We only have two possibilities and so already two trials would make a success “expectable”. But this is false in a sense because it is very well possible that we bet on head and we flip two tails. So this expectability was indeed just expectability and not certainty. And you may say, of course, here nothing is certain. But now comes the point what the Law Of Big Numbers claims, namely that after a while the head must happen. So infinite many failures are impossible. This is not only accepted by everybody but some even turned it into a gambling strategy. Indeed, all you have to do, always double your bets and you will win for sure! So you put one dollar on head. If you win you walk away laughing with your two dollars pay out which means one dollar profit. If you loose, you place next two dollars on head. If you win now then you get four dollars and you only spent three so you again won a dollar. If you loose and so you already lost three dollars, then you bet next four on head. So you spent seven dollars. But again if you win, you get eight, so you will be one dollar ahead.
For a child this is a very good game but gamblers would never use this simply because tails can repeat easily even ten times. Now, by our method this would require two times two times two times . . . repeated ten times which is 1024 dollars.
Indeed, observe the doubles 2 , 4 , 8 , 16 , 32 , 64 , 128 , 256 , 512 , 1024.
Returning to real math, the Law Of Big Numbers merely says that something happens before the end of time and thus seems pretty useless itself. But now we can come to the second application of coin flips that brings the lottery a bit closer, namely that we can regard not just the individual flips but whole consecutive segments.
We all feel that two coins landing on heads both is not half chanced any more rather only a quarter. If you are a formalist by nature then you might justify this by saying that half times half is a quarter. But if you are a thinker than you can get a more realistic explanation. Indeed, now we have not two possible outcomes rather four, namely: 00 , 01 , 10 , 11. So, lets forget this multiplying of chances and rather go for longer segments. Can we get a hundred consecutive heads? Of course we can and an even better visualization of this is not to try a coin hundred times rather throw hundred coins simultaneously. It’s a lot of coin to throw but we are mind experimenting anyway! Now throwing up the hundred coins from a hat is a single trial. To get all heads has some very small non zero chance. So by the Law Of Big Numbers if you repeat this throwing a hat full of coins enough many times, once all the hundred coins will land on heads. Probably this would take billions of years but it would happen once.
Now we go back again from the simultaneous throw, that is using a hat full of coins, to using a single coin and watch for the consecutive hundred segments. This would

slow down the experiment and turn the billion years to even more but we have a point here. In fact, we have two! The first is more like a side track by realizing that actually the consecutive hundred segments is not quite precise. Indeed, should we always look at the next hundred fix windows or continually move the window ahead in the written down results? The throwings from a hat would mean the first but this second could bring in hundred consecutive segments mixed from the two widows after each other. So actually this increases our chances and so we weren't quite right that this single coin usage is a waste of time. But this not the important point.
The really big advantage of going back from simultaneous throwings to consecutive, is that these will give a single infinite sequence. So then the claim of the Law Of Big Numbers, that there will appear a hundred long window with all heads, will at once mean that actually infinite many such will appear. Indeed, once one appears, we can regard the rest of the sequence as a new one, where the law again claims one, and so on. And again the windows could be meant in two sense! Placed after each other or moved step by step. Placed after each other we might miss some segments that come about overlappingly but as we see it doesn't matter! Even this more rigid looking for success will happen infinitely often.
I already said that we are actually playing here with billions to the billons years as realities but lets play a bit even further! So, now I side track a bit again and regard instead of coin throws, random hits to the keyboard of a typewriter. If you are that young then "typewriter" is an old fashioned computer keyboard that writes directly to a sheet of paper. This was a problem because if you didn't look, you could easily write off the paper. Luckily, there were margin settings even on these "ancient" machines and actually a bell rang to tell you to move the so called carriage back and start the next line. So this is the "enter" key of the present and later on electric typewriters indeed a single key did this. This side track could be better applied for computer keyboard or electric typewriter but the original version was formulated by Borel for old typewriters and is called Borel's Monkey. The reason is simply that we imagine that a monkey is punching the keyboard to make it random.
Now if we ignore the line endings, that is we imagine an infinite text typed in a single line or we use electric typewriter where the monkey can do the "enter" or new line key too, plus we imagine an infinite long paper, then the monkey will type for ever.
In the infinite sequence of symbols punched by the monkey, again we can visualise arbitrary long consecutive windows and in those windows every possible letter, comma, space, enter and so on combinations will appear, in fact infinitely often.
For example there has to be a billion long segment with all commas.
More sacrilegiously, we can regard windows with the exact length of a given version of the Bible, say the King James' version. Thus, this will also appear infinitely often in the text that the monkey types down accidentally!
Now that you have a feel how big is an infinite sequence we can return to 0-s and 1-s.

The most important segments in the sequences are the beginnings! These will give us the roads to exactly tell what differentiate a random sequence from a manipulated one that shows some strangeness. But as we just saw, even random sequences have some weird or strange features, so we have to be careful what we should really declare as strange. Of course, these were segment appearances not beginnings but then we could think that this is even worse. The beginnings are irrelevant! Any beginning is possible in some random sequence exactly by the Law Of Big Numbers. Indeed, we can start a sequence and wait until the required beginning as segment appears, Then we simply ignore the useless trials before and voila we obtained a random sequence that starts as

we prescribed. This was an interesting argument, so obviously we can not claim singular beginning appearances as strangenesses, and we didn't mean that either.
But finite many beginnings is again merely some beginning, so the real solution will be to claim infinite many certain beginnings.
The simplest and also visually easiest claim that we can make about looking at a beginning is whether the 0-s or the 1-s are in excess. Of course for even long beginnings they could be exactly the same. This seems rather rare occurrence but to show how wrong we are, lets just first check out the exact number of possibilities for small even lengths.
For 2 length, we have altogether 4 possible beginnings: 00 , 01 , 10 , 11.
From these, 2 obey our property, being the 0-s and 1-s same in number, namely: 01 and 10.
For the next even number length 4 , we have 16 possible beginnings:
0000 , 0001 , 0010 , 0011 , 0100 , 0101 , 0110 , 0111 , 1000 , 1001 , 1010 , 1011 , 1100 , 1101 , 1110 , 1111.
6 of these have equal number of 0-s and 1-s , namely:
0011 , 0101 , 0110 , 1001 , 1010 , 1100.
Next comes 6 length which has 64 possible beginnings.
Indeed, every additional length doubles the number of possible beginnings because each earlier can be continued two ways. So, 5 length has two times 16 that is 32 possibilities and then 6 length makes double again that is 64.
Now we have 20 beginnings that have equal many 0-s and 1-s:

000 111	,	001 011	,	011 001	,	111 000
		010 101		101 010		
		100 110		110 100		

Here I went by breaking up the 6 length into two 3 long first and second half.
This way the 0 , 1 occurrence number in the first half tell this in the second too.
The combinations under each other can all be chosen and so in those two groups we have 3 times 3 that is 9 possibilities and so altogether we indeed have:
1 + 9 + 9 + 1 = 20.
This same method gives for the next length 8 , from the total 256 possibilities the following good ones:

0000 1111	,	0001 0111	,	0011 0011	,	0111 0001	,	1111 0000
		0010 1011		0101 0101		1011 0010		
		0100 1101		0110 0110		1101 0100		
		1000 1110		1001 1001		1110 1000		
				1010 1010				
				1100 1100				

So in number : 1 + 16 + 36 + 16 + 1 = 70

So, the ratios of the beginnings with equal numbered 0-s and 1-s were:

$$\frac{2}{4} , \frac{6}{16} , \frac{20}{64} , \frac{70}{256} \quad \text{or simplified:} \quad \frac{1}{2} , \frac{1}{4} , \frac{5}{16} , \frac{35}{128}$$

These ratios are actually the chances of our property for the particular lengths.

For example, trying out 8 long segments, we should get about 35 such perfectly equal 0 and 1 situations from every tried 128 segments. It's quite a lot!
Now we will again look at even long beginnings but the property we regard is that this even long beginning is exactly repeating its first half.

From the 2 long 00 , 01 , 10 , 11 beginnings again two are okay: 00 and 11.
From the 16 many 4 long ones, the self repeating ones are :
0000 , 0101 , 1010 , 1111

From the 64 many 6 long ones the repeating ones are:

000000 , 001001 , 010010 , 100100 , 011011 , 101101 , 110110 , 111111.

As we see, the number of the repeating ones always doubles and that's logical because they are simply the same as all half length possibilities.
But as we saw the total possibilities multiply by four.
So the ratios are now:

$$\frac{2}{4} \;,\; \frac{4}{16} \;,\; \frac{8}{64} \;,\; \frac{16}{256} \quad \text{or simplified:} \quad \frac{1}{2} \;,\; \frac{1}{4} \;,\; \frac{1}{8} \;,\; \frac{1}{16}$$

These always halved values have a famous property!
Namely, that if we add up the values then the last member added once more would make the total into 1. This can be seen by starting from the trivial case of $\frac{1}{2}$ and then gradually replace the last second member by two of its own halves:

$$1 = \frac{1}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \;...$$

This not only shows that these halved values if added up always remain under 1 but that they get arbitrary close to 1 so their total sum is actually 1.

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \;... \quad = 1$$

Visually of course it is even simpler as moving to a wall that is a meter away from us. We simply have to go always half way. So we move first half meter then a quarter and so on. The remaining distances are also halved and are half, quarter and so on.
This fact that infinite many numbers can add up to merely a finite value was turned into the famous Achilles paradox by regarding it not just in space but in time and add a few twists or smoke and mirrors. Indeed, the simple transition to time is not enough to make a paradox because today we have enough abstraction to visualize quite well that the previous moving to the wall is a similarly infinitely addable in time too.
If we move with a fix speed then since we move less and less distances, we obviously need less and less time too. So we simply approach the arrival time point also by approaching time points. This is not paradoxical. There are always infinite many times before any "now". Namely a minute ago, half minute ago a quarter minute ago and so on.
Of course, if we don't move with fix speed rather spend same time say a second for every new half motion forward, then we never reach the wall because infinite many seconds would still take for ever.

The Greeks were smart! They mixed up this situation with the normal kept speed by simply not approaching the wall rather somebody moving in front of us.
Now if we are faster, then no matter how ahead is someone in front of us, we will catch up. But the Greek philosophers said this: Suppose you catch up to the point where the chased person was at the start. By this time he moved too. Then when you catch this point, he will be ahead again. And so on, you never catch him.
These catching ups are happening in shorter and shorter times and add up to the exact time of catching him. Being behind infinite many times was true but infinite many times being behind doesn't mean "for ever".

The original paradox was formulated for Achilles the famous runner and I intentionally neglected the style in favour of the meanings.
And yet, I myself call a new paradox the "Anti Achilles" paradox.
This comes about by eliminating the false idea as if the previous adding up of infinitely many smaller and smaller distances would always have to add up to a finite length. No! Smaller and smaller values can add up to infinity and first we are quite clueless how this can happen. The easiest is to start with infinite many equal 1 values. These are obviously infinite in total but they are not decreasing. To make them decreasing is easy by taking of arbitrary small amounts from them, namely we could take of the previously used $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ . \ . \ . \ = 1$ distances and this being only 1 in total would leave the infinity:

$$(1 + 1 + 1 + \ . \ . \ . \ ) - (\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ . \ . \ . \ ) =$$

$$1 - \frac{1}{2} + 1 - \frac{1}{4} + 1 - \frac{1}{8} + 1 - \frac{1}{16} + \ . \ . \ . \ =$$

$$\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ . \ . \ . \ = \infty - 1 = \infty$$

An alternative idea is not to subtract from the equal members rather distribute them into more and more pieces thus getting smaller and smaller members.
Here it's better to start not with infinite many 1-s rather $\frac{1}{2}$ values and then cut the second into two, the third into four, then eight and so equal parts:

$$\infty = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ . \ . \ . \ . \ . \ =.$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \ . \ . \ . \ . \ . \ = \infty .$$

We proved our point because the members are diminishing but we might feel a bit cheated because the members are repeating too.
So now lets increase our sum a tiny bit even further and thus still get ∞ as follows:
We increase the first $\frac{1}{4}$ to $\frac{1}{3}$ but leave the second.

Then we increase the first $\frac{1}{8}$ to $\frac{1}{5}$ the second one to $\frac{1}{6}$ and the third one to $\frac{1}{7}$ but again we leave the last $\frac{1}{8}$.

The same trick can be used for the next $\frac{1}{16}$ group by increasing them starting to $\frac{1}{9}$ then to $\frac{1}{10}$ and so on with finishing leaving $\frac{1}{16}$.

Then these increases give exactly all the reciprocals and thus amazingly:

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \; . \; . \; . \; . \; . \; = \infty .$$

These reciprocals are really quite “small” values and yet their sum is infinite!
We have to start thinking in tendencies of the values not in their individual values.
These reciprocals are small but they are pretty steady! Don’t decrease fast.
The halving fractions were decreasing much faster and that made them finite in total!
This reveals that the actual sum value is not really relevant. And indeed, we could increase the beginning members as we wish, thus adding big values to the sum, yet this wouldn’t change the real tendency of the numbers. This shows an amazing similarity with randomness where also the beginning is irrelevant.
This fact that the total sum being finite or infinite should be the real decider whether the numbers themselves are diminishing fast or slow has an even more convincing twist. This fact will show even more that the sum value is irrelevant.
Think of what happens when in an infinite sum you are not looking at the total value, rather the total left over value after going further and further in the sum. Since these beginning sums are approaching the total, the more you take off the less remains. So actually the remaining ones always have to become arbitrary small, regardless the total value. So the finite valued sums are actually the sums with diminishing left over values. Quite contrary, if the sum has an infinite total then no matter how far you go, the left over remains infinite. Now we’re cooking!
The repeating beginning chances are obviously fast diminishing. They have 1 total.
What about the other chance values we calculated for the even beginnings with exactly equal 0-s and 1-s? It was diminishing visibly slower but is it really slow, that is adding up to infinity? Luckily we don’t have to go into concrete calculations of infinite sums to prove this! A seemingly completely different direction will reveal this fact to us. This direction is the first step towards defining randomness!
Up until now we just assumed that the randomness of an infinite sequence would tell us how it should behave. Namely, the Law Of Big Numbers can be used for its segments. Simply because a random sequence is random from any point further too.
This principle revealed us what could happen by Borel’s Monkey and though it was shocking, we can accept it after we think it through. Now, the surprising two facts are that this principle is not telling everything that is true about random sequences and we have very strong intuitions about such extra features though we are not aware of it. Even more surprising is that these intuitions are about how beginnings must behave!
When I started to look at the beginning properties back in high school, the first one I considered was exactly the mentioned self repetition. We saw how easy it comes out that the chances of this are always halving very fast diminishing values. But I also felt quite independently of this result that this property simply must stop occurring in a random sequence. To repeat newer and newer, longer and longer full beginnings is simply impossible. I even tried actual coin flips and checked how many times could

such accidental repeat happen and it was always very small. Even if we allow overlapping repeats using already failing even long beginning. For example in the following beginning of a random sequence we have three such repeats:
0100101001001010... Indeed, the 3 and 5 and 8 long beginnings repeat.
I was showing this segment to my family members and first asked them if they see anything special. Of course they didn't and after I showed the three repeats then I could ask the real question: Could such repeats go on forever?
To my greatest satisfaction they all said that it can not happen infinitely.
We have an intuition about that this repetition must stop.
But this good news soon turned sour because my mind told me that this stopping somehow must be related to the chance values alone and not them being beginnings.
To demonstrate the difference is actually quite easy by simply regarding again an infinite random sequence but now observing not the even long beginnings rather the even long increasing segments after each other. So look at the first two digits, then the following four, then the next six then the eight, and so on. So we look at these infinite many longer and longer windows and then we can again ask if infinite many will be self repeating ones. If only the chances decide the stopping then here again we must have only finite many occurrences. But if you ask this from naïve observers then you wont get the same definite reply as for the beginnings. This of course makes perfect sense because the beginnings are the random history of the sequence. To repeat a long history is defying the freedom of the future. With shifted repetitions in consecutive segments, we don't get into direct contradiction with the future freedom. That's why we were able to swallow Borel's Monkey too. So what's going on?
Is infinite many repeat possible in the increasing segments?
Obviously, infinite many self repeats will be observable if we are allowed to choose windows freely because a self repeat is just like the Bible (to be sacrilegious again). Observe that these arbitrary long windows come about by simply regarding arbitrary long but fix positioned windows first and then going far away among these.
Here with the fix increasing consecutive even windows we at once predetermine all the locations of the increasing windows. This is what makes it impossible to repeat infinitely the unlikely chanced property. So, in fixed length windows everything is possible and we can regard these for arbitrary cases thus obtaining very weird long windows. But to fix longer and longer windows will not allow everything in them.
The really amazing thing for me today is how I was believing in the importance of everyday intuitions already back then. I had no idea about the existence of a future Didactical Logic which I am certain today. I had no idea that space and time are just material constrains and understanding is the predetermined order of the universe before matter, which again I am certain today. And finally, I had no idea that we humans are in a war to fight out from the ego driven stupidities and metaphysical bullshits the simple common sense truths accessible to everyone. This equality driven true leftism has to be an honest idealism. This is the only road to God and for humanity to enter kindergarten first and encounter other intelligences. Not in the space of matter but through the space we always traveled already.
So how can we see that our intuition about the stopping of the self repeating beginnings are correct, in a way that reveals also the less certain stopping in the increasing even segments too?
We obviously have to start with simpler intuitions!
Our segment intuitions were actually an "and" intuition because we required that a finite many coin flips come out first as this then this then this and so on. But simultaneously they are just appearing together which is exactly the "and".
The other simplest connection of claims is the "or". Thinking in alternatives.

The chances of the “and”-s were quite obvious. The continuation of a segment with a new member allowed doubling the total cases. This means that the chances multiply.
For example one coin has two cases and one favorable. Two coins have four cases 00 , 01 , 10 , 11 but again only one favorable for “and” namely 00.
So indeed, the half chances multiplied to give the chance of the “and” as a quarter.
This was true because the different coins are independent they are free to become random. If the two coins are tied with a tiny rope then throwing them together will alter the “and” outcomes. The rope might decrease or increase the occurrence of two heads together.
We might think that this independence is the most natural requirement of experiments and it should be always part of our assumptions. But this is not so and our strongest intuition about the stopping of the repeat beginnings was based on exactly dependence. Indeed, the newer and newer beginnings must continue each other. Only one extra new coin is tried not the whole longer beginning from the start. Quite on the contrary, the seemingly artificial increasing even segments were such true independent testings of the self repeats.
So things are getting complicated but lets get back to the “or”-s.
The simplest “or” situation is the two outcome possibility of the coin.
This is not just “or” but rather “either-or”.
A bit more interesting is the six sides of a dice. Here we have six alternative cases.
The “either-or” could also be called exclusive “or” because the cases exclude each other and so we claim exactly one of the cases, while the normal “or” is actually claiming only at least one outcome, but allows more too. A coin or a dice is of course exclusive, they can only land on one sides.
We can combine some exclusive cases into so called events with “or”.
So for example, to throw an even number with a dice is the event : 2 or 4 or 6.
A simple thing shows how this “or” combination became more important than the “and”. Namely, that here the set or collection concept became used for the “or”.
So the even throwing event of the dice is simplified as $\{ 2 , 4 , 6 \}$.
The hidden assumption is also that these “or”-s are “either-or”-s.
A different event could be $\{ 1 , 2 , 3 \}$ that is to throw small values.
Now we can combine events with “and” or “or” too and then the cases simply are the common or combined ones.
To throw a small and even value is the single 2 outcome because this is the only common element of $\{ 1 , 2 , 3 \}$ and $\{ 2 , 4 , 6 \}$.
To throw a small or even value is $\{ 1 , 2 , 3 \}$ or $\{ 2 , 4 , 6 \} = \{ 1 , 2 , 3 , 4 , 6 \}$.
These are all trivial formalities that are unimportant!
The real importance starts by thinking about how the chances of “or”-s should come about. Since the “and” was multiplication, we might think that here addition happens.
But this is false and this falsity the start of the real points!
Two coins thrown without tying ropes or magnets is independence and the “and”, that is two heads, that is 00 had a quarter chance. Now the “or” is throwing at least one head and it will not have the chance of the sum that is $\frac{1}{2} + \frac{1}{2} = 1$.
Indeed, the total cases are 00 , 01 , 10 , 11 and the favorable ones ore 00 , 10 , 10.
So the chance is $\frac{3}{4}$.
The impossibility of the formally obtained 1 value could be also argued as that being certainty and realizing that two throws can not guarantee a sure head.
Throwing even a billion coins we can not get a sure head because of the very unlikely but possible event of all landing on tails.

This suggests that these “or” chances should be approached as opposites of “and”-s. Indeed, at least one head means exactly the opposite of all being tails.

The chance of this all tails from hundred throws is $\frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \ . \ . \ . \ = \frac{1}{2^{100}}$.

And so the chance of at least one heads is $1 - \frac{1}{2^{100}}$.

To check the more realistic two coins scenario, here the all tails have $\frac{1}{2} \bullet \frac{1}{2} = \frac{1}{4}$ chance and the at least one head is indeed $1 - \frac{1}{4} = \frac{3}{4}$.

So the false addition assumption was not only false but seemingly also unnecessary because this tricky double negation method, that is taking the 1 minus value of the “and”-s of the opposite outcomes, that is the product of the 1 minus values is perfect.

If some **1** , **2** , . . . , **n** events are independent, then:

$$p(\mathbf{1} \text{ or } \mathbf{2} \text{ or } \ldots \text{ or } \mathbf{n}) = 1 - (1-p_1)(1-p_2) \ldots (1-p_n)$$

But now comes the even more interesting part that the addition is still not useless!

Our didactical mistake was that we looked at independent events even though we saw that it was the special one for “and”-s and the “or”-s had their special ones too namely the exclusion. The landing of two coins are independent but not exclusive. Exactly the independence denies the exclusions, they can come about any ways.

The six sides of the dice or two sides of the coin are indeed exclusive and here indeed the addition will be the correct. For a single coin the head or tail is $\frac{1}{2} + \frac{1}{2} = 1$.

And indeed it has to land on one of them.

So, for exclusive events the addition of the chances is correct!

But what was the use of this, you may ask if we never really use such trivial events?

The real use is that we can use this fact for “or”-ing not just cases but events too.

Events in general could be exclusive though indeed they are rarely are. But the point is that this exclusion is the situation with the highest chance of an “or” outcome.

All realistic events if are “or” connected must give a chance value being maximum the sum of the individual chances. And this is now true for all events even if they are not independent. So : $p(\mathbf{1} \text{ or } \mathbf{2} \text{ or } \ldots \text{ or } \mathbf{n}) \le p_1 + p_2 + \ldots + p_n$.

But as an interesting special case it has to stand for the independent events too.

There we had our double negated formula already, which means that we must have also the inequality: $1 - (1-p_1)(1-p_2) \ldots (1-p_n) \le p_1 + p_2 + \ldots + p_n$

And indeed this can be proved algebraically too, not regarding chances at all.

We still might think that this sum bounding of the “or” chances is so rough that it couldn’t lead to anything important. That’s where we are wrong. This bounding will show us quite easily why the self repeating beginnings must stop!

Remember that the fast diminishing was accepted as having a finite sum and such finite sum means that the left over sum from any point is diminishing.

So, fast diminishing is simply diminishing of the left over totals.

The dynamic false impressions are static!

Infinite many values in a big hat are either finite or infinite in their total.

The finiteness means that picking out values, the left over total has to become arbitrary small.

Instead of values being in a hat we can imagine objects in the hat that have their chances as values. Namely, these objects could be beginnings.

The attached chance values are $\frac{1}{2^n}$ for any n long beginning.
But observe that we obtained these chances taken from the fix n lengths.
Indeed, there are $2 \bullet 2 \bullet \ldots \bullet 2 = 2^n$ many different beginning with n length and every single one has the same chance and thus exactly $\frac{1}{2^n}$.
Throwing beginnings with different lengths into one hat is then confusing because these attached chances are not chances in the hat.
That chance from the hat is actually not even defined!
To find a chance value one has to be able to repeat things.
To pick out elements, that is new beginnings from a hat is meaningful.
But these are not repeatable picks and not comparable.
In spite of this, for using longer and longer beginnings from our hat we can use these chances meaningfully too if we regard this continuation as the continuing length.
Among a length, these chance values are indeed the chances of possible continuation.
The chance of finding a self repeating beginning from the hundred to hundred fifty lengths is exactly the "or" connection of these fifty many length groups.
So then by the bounding of the "or"-s, the chance of at least one occurrence has to be maximum the sum of those fifty chances. Stepping to infinite many of these groups:
The chance of finding at least one self repeating beginning that is longer than hundred, is bounded by the total of the chances after hundred length.
But these values being finite in total mean that that these bounds if we go from hundred to thousand and more, have to diminish.
Since these bounds diminish, thus the actual chances of the at least one occurrences are diminishing too. That's exciting but still nothing tangible!
Now comes the first crucial abstraction step!
We formulate a claim! Infinite many occurrence!
This obviously implies each of the at least one occurrences after any particular length.
Now comes the second crucial abstraction!
If some principal event always implies an other then the chance of the implied event can not be bigger than the principal. Indeed, in whatever method we check the favorable cases the principal will always bring about the consequence.
Our principal event is having infinite many self repeating beginnings and it indeed causes now an infinity of implied or consequence events, namely having at least one occurrence after any particular length.
The chance of the principal event can not be more than any of these.
But these are now diminishing and the only number that is not more than any of some diminishing numbers, is zero.
So the chance of infinite occurrence is zero.
Now comes the third abstraction, we regard this zero chance as impossibility in a random sequence.
The zero chance itself obviously can not mean impossibility in general because we can manufacture easily infinitely self repeating sequences.
But even regarding only the random sequences, the zero chance as impossibility is still a big step!
To see this, we have to realize that there are random outcomes that have zero chance and yet are not impossible!
A dart board has infinite many points on it. The dart is landing in one single point.
(If we at least don't miss completely the whole board.)
So, here every single outcome is actually a zero chanced case happening and so possible.

At the coin flips, the original elemental cases were half chanced much simpler cases. But from these we created the infinite sequences as new elemental cases and now among these we seem to claim that zero chance means impossibility.
The crucial difference is that now we didn't use this for individual sequences.
The zero chanced infinitely repeating beginnings is not one sequence.
It's all those that we can manufacture to be like that and even those that we can not manufacture because are not determined. These have randomness in them, lot of outcomes that we can not predict but these sequences are still not random as whole because this property itself was happening.
Of course, we can imagine a single random sequence and require instead of the self repetition that the beginnings must exactly go as in this chosen random sequence.
This as a beginning property would be even more narrow or diminishing in the possible continuations as the self repetition. Indeed, only one possibility is in every length and so the total chances are: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 1$
Seemingly, the same values as we had for self repetitions but those were only among the even lengths, so though the total is the same, this is indeed "faster" diminishing.
The point is that if we accept the finite total as the measure of critical narrowness, where random sequences can not go through, then we must conclude that no random sequence could go infinitely among these beginnings either. They have to stop.
And yet the random sequence we started with, does obey itself, never stops!
Amazingly, a similar contradiction can even be raised for the dart board.
Even though there the zero chanced individualities were the cases, if we pick a point then to hit exactly there would be truly impossible! But this unpredictability of the individual cases from an infinite continuous set, is not dealt by any randomness theory at the present. On the other hand, the exclusion of individual random sequences is more a problem with the theory that we are trying to formulate now.

If we regard the beginning properties naively as I did when I was sixteen then there is no problem! A picked random sequence is simply not a property.
If however we want to regard arbitrary collections of beginnings then we are in trouble because we could simply put all the beginnings of a fix random sequence into our hat too.
There are two ways out of this dilemma.
One is to improve the naïve property concept and thus only accept beginning selections that are given explicitly in a formal system. Such system must be built from finite many basic concepts and so the use of a fix random sequence to select beginnings is excluded. Then we can realize that the whole trouble of introducing a system is avoidable. After all, the only crucial use of this was to avoid a random sequence because the finite many data of the system could not contain the sequence.
So then why don't we just allow any listings of beginnings without any formal system to formulate properties. All we need is that this listing of our infinite many beginnings is determined by some finite set of rules. In short, a computer should tell the beginnings that are the chosen ones. Indeed, any old fashioned property that we can create can be easily mechanized. Now it's not even necessary that we generate the chosen beginnings in increasing lengths. We might create first longer ones and only then decide to select shorter ones. So this computer is not a progressively predicting machine at all. It is simply an exclusion of individual random selections.
Every computer program that generates beginnings, that is binary segments, is a perfect replacement of our naïve property concept!

If the total chance values of the generated segments is finite then this computer program is not just a collector but actually a strangeness collector.
Namely, for an infinite sequence to be able to grow by these segments as continuing beginnings would be a strangeness. No random sequence could do that! It will have maybe some beginnings from the computer list but only finite many. It must stop.

A random sequence must stop from beginning options given by a computer if the options are narrow enough, that is have a finite total chance.

So we have three finiteness in our randomness:

The qualitative finiteness is the use of computer with a particular finite program that determines the possible beginnings.

The quantitative finiteness is the critical narrowness. The chance total must be finite.

The obeying finiteness means that a random sequence can only have finite may beginnings from the list that itself was finite in the previous two sense.

To see how important is the quantitative finiteness condition, that is how easily could a computer generate beginnings that allow random continuations, here is an example:

0 , 1 , 00 , 01 , 10 , 11 , 000 , 001 , 010 , 011 , 100 , 101 , 110 , 111, 0000 , 0001 , . . .

We listed all possible beginnings and so every sequence is buildable from this list.
This set of course has an infinite chance total, namely it is:

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \; . \; . \; . \; . \; = \infty .$$

The random sequences are simply those that fail in all possible computer generated beginning collections that are critically narrow, that is finite chance totaled.

A computer that generates the beginnings can easily calculate and even add up their chance values too. Unfortunately, the generation up to a point would only give the obtained total at that point, so we could not tell whether the quantitative condition will stand or not. Only being smarter than the machine and telling in advance the finite total could give practical strangenesses. This means that either we use properties for which we know mathematical methods to prove the finiteness or we use a machine program but we can look into the program and establish in advance that the generated total will be finite.
At the self repeating beginnings for example it was the easy chance calculation group by group that gave us instantly that the total will be finite. But in our other example we were not so lucky. We gave a method to generate group by group all the equal 0 and 1 beginnings but the final formula would be still quite complicated.
So, even the naïve property approach doesn’t necessarily mean a success.
Luckily, the randomness concept itself can help to avoid proving an infinite sum.
We will show that a random sequence could not stop in the property of having equal 0-s and 1-s. This of course implies that the property chance total is infinite.

The crucial first thing to realize is that that the opposite of a beginning property is also a beginning property. The second is that even though the negative of the claim to stop in a property, is simply to continue in it, regarding the negative property we get something much more! Indeed, to stop in a property does not merely imply that we have to continue in the negative but that we have to “stay” in there from a point.
Using our concrete case, if a random sequence would stop in the equalization property then it would have to stay in the negation that is in the property of having non equal 0-s and 1-s. But this would mean that actually only one of these would have to stay.
Indeed, otherwise, that is both occurring infinite many times, the one by one alteration of the difference in their numbers would have to bring about infinite many equalizations too. Of course, to have more 0-s or 1-s for ever, feels impossible and so we could easily reject the whole possibility.
To fully appreciate what just happened, lets regard a new beginning property:

We look at how many times the last digit of the property is repeating there and if this is more than any earlier number of repeats of the same digit then the property stands.
The last digit just achieved a record of its repetition.

For example, the following beginning 001110100110011111 qualifies because the last digit 1 finished five times and earlier it only repeated three times.
We feel that this is pretty rare occurrence and yet we can show at once that it has to happen infinitely in a random sequence and thus also the chance values of this property must be infinite in total.
Indeed, since there are longer and longer full 1 segments in any random sequence, we can find the first of these for any length. Before this the repetitions are less and so these segments added to the beginnings before them and regarded as new beginnings will provide these properties with arbitrary long end repeats.

Again, simple segment consideration using the Law Of Big Numbers mixed with the classical usage of infinites implied a totally different result about the chances being infinite in total. If we accept that our randomness definition was correct.

This was the forgotten promising direction that my mind almost wiped out, though at that time I felt that it must mean something very important.
This wipe out is understandable if we look into how many other promising directions and problems are lurking behind the seemingly simple final randomness concept.
As I said, I don’t want to diverse into these, rather formulate the much more important conclusion of this wiped out direction. But just to give you a feel of those other important things, I will just list the most important points:

1.)
First, I will return to my claim that the Law Of Large Numbers is a deception.
It claims that the 0-s and the 1-s both have to become half half in the long runs.
The fact that we have arbitrary long full 0-s and 1-s , combined with the fact that we have infinite many equalizations, can easily show that we also will have arbitrary big differences in the number of 0-s and 1-s in the beginnings.
In short, arbitrary big oscillations of their totals from the start will come about.
So the intuitive half half expression was totally unjustified.

Of course, the "fine print" of the Law Of Large Numbers says that we have to divide the number of the 0 or 1 occurrences with the total length of the beginning and then this fraction is that has to approach $\frac{1}{2}$.

So $\frac{n}{N} \rightarrow \frac{1}{2}$ where n is the number of 0-s or 1-s and N is the total length.

This of course means $|\frac{n}{N} - \frac{1}{2}| \rightarrow 0$. So in fact, an approach to 0 or diminishing.

We used this expression many times but we didn't actually dissected it.
What does it really mean?

It means that $|\frac{n}{N} - \frac{1}{2}|$ remains under arbitrary small ε value from a point.

That is: For every ε there is an M that if $N > M$ then $|\frac{n}{N} - \frac{1}{2}| < \varepsilon$.

In other words, for every ε there is an M from where $|\frac{n}{N} - \frac{1}{2}| < \varepsilon$ remains true, that is it stays. But staying is a stopping for the negated property $|\frac{n}{N} - \frac{1}{2}| \geq \varepsilon$.

So, with this we get that for every ε there is an M that $|\frac{n}{N} - \frac{1}{2}| \geq \varepsilon$ is never true above M and so quite simply it stops in N that is for the beginnings.
This of course is true for every ε, so actually we have an infinity of properties, for every possible ε values but each must stop for N that is for beginnings.
So, we see crystal clearly that the new definition of randomness works and replaces the Law Of Large Numbers that claimed a seemingly infinite tendency for random sequences. The correct replacement is a stopping as it always has to be.

The random sequences must stop in all the $|\frac{n}{N} - \frac{1}{2}| \geq \varepsilon$ properties.

To prove that indeed these properties are narrow, that is have finite chance totals, would be much harder.

2.)
So, we got again some beginning propery where the establishing of the total's finiteness or infinity is hard. And this brings in the second vision. This could be called the "Beginning Exclusion" vision. Its basic idea could be approached from a failing situation. Namely, that infinite minus infinite can be still infinite, doesn't have to be finite. Indeed, this is a failing to establish that a chance sum of some beginning set is finite from merely that the left over complement set's total is infinite. They can be both infinite and these intuitively mean properties that both occur infinitely in the random sequences. They are not stop and stay pairs, rather alternate. So, this is a natural reason of why the "failure" happened. And yet, we can push the infinite minus infinite idea into a right track if we apply it in groups that approach the total set.
The simplest is to regard the same length groups, like in beginning properties and look how many wrong ones are outside, excluded. If this is a lot, then the allowed ones can be established to diminish fast to a finite total. An other way of looking is that the exclusion can gave a left over too, as a "cover" for our original set.
Usually this "cover" aspect is emphasized in this vision and it leads to "measure".
But the exclusion is a vital original idea behind and I will show this by starting with the most elemental case that is actually a paradox. I call it the Exclusion Paradox.
Its resolution is the Proportional Narrowing Plausibility.

We start by asking someone how many numbers are up to a million that don't contain any 7 digit. Since there are ten possible digits, we tend to believe that the number of the 7-less numbers can not be that small, maybe about eighty percent.
Up to ten we indeed have ninety percent of the numbers that are 7-less.
Then up to hundred we at once have a group of numbers namely the 7 starting "seventies" that are all out and these are ten percent so leave ninety percent. But among these in each ten groups there are the 7 ending ones and so we have again only ninety percent remaining from those. Thus ninety percent of ninety percent that is $.9 \bullet .9 = .81$ that is 81 percent is 7-less.
Up to thousand we have the 7 hundreds then the seventies and finally again the 7 ending ones. So now the percent is $.9 \bullet .9 \bullet .9 = .729$ so only 73 percent.
By a million, the product goes under $.5$ so we have less than half of the numbers.
Going to infinite decimals, this means that the ones that don't contain 7 for a longer and longer beginning are proportionally less and less amounts. The chance of the beginning property of not containing 7 is diminishing proportionally.
Such proportional diminishing is first of all indeed diminishing, that is $p^n \to 0$.
Of course, only if p is under 1. The trick to see this is that in this case p can be written as $\frac{1}{1+\varepsilon}$ with some positive ε value and then:

$$\left(\frac{1}{1+\varepsilon}\right)^n = \frac{1}{(1+\varepsilon)^n} = \frac{1}{1+n\,\varepsilon+\ldots} < \frac{1}{1+n\,\varepsilon}$$ which can be made arbitrary small.

The decrease of p^n even feels fast. But is it fast by our formal earlier sense as having finite sum, that is $p + p^2 + p^3 + \ . \ . \ . \ =$ finite ?.
The trick to see this is to realize that multiplying our sum with p would actually give the same without the first member:

$$(p + p^2 + p^3 + \ . \ . \ . \ )\,p = p^2 + p^3 + \ . \ . \ . \ = (p + p^2 + p^3 + \ . \ . \ . \ ) - p.$$

So, taking the left side to the right and p from the right to the left:

$$p = (p + p^2 + p^3 + \ . \ . \ ) - (p + p^2 + p^3 + \ . \ . \ )\,p = (p + p^2 + p^3 + \ . \ . \ )(1 - p)$$

And so $p + p^2 + p^3 + \ . \ . \ . \ = \frac{p}{1-p}$.
Of course, this was not quite correct because if the sum is infinite then taking it out as common factor is false. But the idea can be saved if instead we use finite sums.
Then the p multiple gives an extra member and so:

$$(p + p^2 + p^3 + \ . \ . + p^n)\,p = p + p^2 + p^3 + \ . \ . + p^n + p^{n+1} - p$$

So now taking the last two members to the left and the sums to the right:

$p - p^{n+1} = (p + p^2 + p^3 + \ . \ . + p^n)(1 - p)$ but now correctly, and so:

$$p + p^2 + p^3 + \ . \ . + p^n = \frac{p - p^{n+1}}{1-p}$$

This indeed approaches $\frac{p}{1-p}$ because the numerator approaches p.
But this algebraic verification of the fast diminishing is not our point here now.

Rather, that we can “see” how the proportionality resolves our paradox.
And in fact, what we see is a leap to much wider exclusions.
In an infinite decimal not to have any 7 digits, must have zero chance by the diminishing chances of not having 7 for all beginnings. Indeed, again the principal fact of not having any 7 implies the not having one in any beginning an so it must have a lower chance than these arbitrary small values and thus must be zero.
But now regarding the decimals as points, the 7 exclusions are intervals excluded.
So we blacken out the seventh of the ten equal decimal intervals corresponding to the first decimal value. Then, in the nine white remaining one tenth intervals we again blacken out the seventh of each of the second ten divisions. Then again and again. The blackenings are only one tenths but are applied always to the remaining white places.
The whole [0 , 1] interval of the number line is becoming black.
Most amazingly, if we would merely forbid the 7 digit at only the odd decimal places, then this situation would remain the same. Indeed, we have a less proportion of blackening now but the remaining white lengths in total would decrease by again in a fix p proportion. So the white length in total is diminishing.
Any infinite many fix proportional exclusions mean infinite many 1 minus that proportional narrowings. And this becomes arbitrary small.
So, on one side we completely ignored the fastness, the finite total and yet we obtained that the infinite behavior seems impossible by simple diminishing.
Well, actually the diminishing was not simple because it still had to be proportional but now at any infinite many times for the beginning groups.
But then we can also argue that if we just know any fix diminishings then we can always combine beginning groups that the diminishings become less than proportional and so again we can imply final length diminishing.
This is a dangerous shift from the narrowness of the total chances as finiteness to the partitioning of the beginnings into groups and require merely diminishing for their chances. We might even jump to find a contradiction in this whole idea by regarding a slowly diminishing situation. And then we see what’s going on.
These diminishings are for the covers of the beginning groups by continuability.
Any beginning set can be covered by a finite length. Indeed the 0 and 1 beginnings for example cover everything because everything is a continuation of one of these.
For the slowly diminishing groups we simply can not find diminishing covers!
The length group values might diminish but they don’t cover! New beginnings appear that are not continuations of earlier ones.
A theoretical concept helps to see deeper. Namely, we can always create continuing covering groups easily. First, we eliminate all those beginnings in our beginning collection that have no continuations at all. Indeed, these are useless to build sequences anyway. Then we can call the minimal beginnings those that are not continuations of anything else in the set. The set of these is our first group. The left over ones are all continuations of these. Among these, the minimals are the ones that have exactly one sub beginning in the set. This is our next group. Then again we regard the minimals and so on. So we split the beginning set into continuing covering sets each containing the beginnings having one more sub beginnings. So then seemingly the new nil covering vision could be applied. That’s where the hidden conditions come in! Not only these groups must be given effectively but their chance totals too. Otherwise we can not select proportionally decreasing sub groups.
This mechanical or “hard” diminishing is not the only hard part. The partitioning to be mechanical would be hard too. Indeed, how do we know if a beginning has exactly a certain number of sub beginnings? The trick that Solovay used was not to regard these groups as splits rather as narrowings.

So we start with the total set of beginnings. Then leave out the minimals, then again the minimals in the remainings and so on. So, now these narrowing groups contain the beginnings that have at least one, at least two and so on sub beginnings. As far as covering goes this means the same. The minimals cover everything and so on. But now the generation of these narrowing beginning sets is easy. The first member is everything generated by our original machine. As our machine realizes that a generated beginning has already one sub beginning generated, it generates this in the second group as well. Then as we see more and more sub beginnings, we generate those in later and later groups too. The proof of that we can effectively tell diminishing totals that cover these group totals follows from the continuabilities.
Namely, the second member will be coverable by maximum half of the original total, the third by maximum a third and so on. Indeed, the exactly n sub beginning containing beginnings cover the at least n ones. So it's enough to see that these exclusive groups have these reciprocal covers. The first n of these must have a cover maximum the full total of the set and they have non increasing covers, so the n-th has a cover maximum n-th of the total. The reversal, that is to get a finite totaling beginning set from diminishing coverings is again relies on the hard diminishing. Only this allows to select a proportionally diminishing, say always halving cover sequence and there we can combine the covers.
Now back to the promised wider exclusion meanings, the first comes about if the infinite many blackenings out are not given concretely, rather conditionally. Namely depending on the beginnings themselves. We can exclude the digit 7 after certain beginnings and these still would diminish the remaining decimals. With binary "decimals" that is with coin flips, this goes back to a relic that goes back to an even older relic which is an other directional widening of the exclusions.
After certain beginnings, telling what comes next 0 or 1 is a clear violation of randomness by our intuitions if this can happen infinite many times. With our new vision now we see that these seemingly rare predictions still reduce the possibilities to zero. But the exact digit prediction is not necessary, it can be flexible too like the places themselves by beginning conditions. Namely, if we know that only "usually" after some beginnings a 0 comes, then we could still win big by betting. So, the original idea of Von Mises was to claim that these infinite many place selections if looked as a new sequence, should obey the Law Of Large Numbers in random sequences. If a sequence wouldn't obey it that is would have such places with altered relative frequency rates, then we could bet on these and it should count as a strangeness. Unfortunately, he couldn't define how to choose these observational sub sequences. Much later, Church figured out the beginning selections by a machine but he still clinged on to Von Mises idea to regard it as observational sub sequence and use the Law Of Large Numbers. Church's definition was faulty! It allowed sequences that didn't obey the Law Of Equalization, that is contained always excess 0-s or 1-s from a point on. Martin Löf's idea of diminishingly coverable beginning sets was the crucial break through. He formally required half, quarter and so on covers and so the whole predictable diminishing or hard diminishing was a bit shoved under the carpet. The finite total of course is even simpler and avoids the whole concept of the cover.
The false obsession with the Law Of Large Numbers didn't die after Martin Löf's breakthrough. Lambalgen created an even worse monstrosity based on Von Mises idea than Church did.
The Law Of Big Numbers is following from any randomness definition.
Indeed, if a prescribed segment wouldn't appear in the widows of a random sequence then this would be like excluding the digit 7 in decimals.

3.)
The negative or complement beginning property or collection was used frequently already but this raises a deep problem. Just because a program can determine the original hat, can we claim that the complement hat is also machine determined?
The original need for computer and program was the avoidance of particular random sequence as template and so why shouldn't we accept the complement too. Just as a concrete random sequence can not obey a machine generation, it can not obey the left over complement either.
So, the complement should be regarded as machine determined.
But apart from this, the question still remains if the complement could be generated by an other machine or not.
The fact that for a complicated enough machine, the complement of the generated set becomes even more complicated, so that definitely and absolutely no machine can generate it, is the crucial fundamental point at the bottom of the famous Gödel Incompleteness Theorem. First seen crystal clearly by Turing.
The more general tendency that "most" machines should have such non generable complements became precise with Rice's Theorem, telling general conditions when the complements are always non generable. Though the proof of this is easy and formally the claim itself can be "melted" into the old results, there is a total jump in plausibilities by the major consequence of the result. Finally, the concept of program has been fully recognized. Indeed, this consequence says that if we regard programs that use their data not merely as dead data but as runnable programs, then the complements are always non generable. Anybody's heart that has a sense of intuitional bravery and honesty to trust himself not just the socially accepted formal junk, will skip a beat even just hearing these facts. And indeed, Rice's Theorem is the most important result of New Math after the Well Ordering Theorem.
(In fact, the Well Ordering Theorem has its own oversimplifications too.)
But there is an external naïve "proof" of that I am right and the epigone little monkeys who derive Rice's Theorem in a flash without mentioning its importance, are wrong!
Indeed, if it's so trivial and "easy", then why did it take twenty years to see it?
Why did the founders, Gödel, Turing and the "Bible writer" Kleene miss it?
Because it is not the derivabilities that tell the truth of mathematics!
It is the visible plausibilities that drive everything, even these derivabilities!

4.)
As we mentioned, the chance total itself is only gradually calculable by the machine.
This with the previous problem leads to something amazing.
The chance total is obviously determined by the machine as abstract determination but it is only gradually obtainable with the running of the machine. As I also mentioned, we might have insight through the running rules, that is from the program and tell the crucial distinction of finite or infinite total. But assuming that we know the total exactly as an infinite decimal value, it would be able to tell more and more of the non generable beginnings, that is the complement too.
This then implies that this total chance value is not just non generable by a machine but can not even have infinitely continuing beginning properties, it has to be random.
And yet is determined by a machine. These are Chaitin's omega numbers.
The earlier mentioned Martin Löf form of the randomness then offers a natural alteration to exclude these omega numbers from randomness. Namely, we merely have to drop the hardness that is machine predictability of the diminishing.
Apart from this, many other alternative and altered randomness definitions were created from Martin Löf's.

These dazzling directions gave you a glimpse of how amazingly intricate is the role of randomness in the fields of New Math already.
And yet, there is a simple straight forward idea missed by all these directions!

The simple segment arguments leading to beginning properties that have to repeat in random sequences was not exploited up until now.
And yet, these easily derivable properties are “practically” the only infinitely occurring beginnings in the random sequences.
The “practically” means that we must ignore beginning properties that can inherit from merely an already existing sub beginning. The beginnings are so small compared to the infinites that they can not influence the infinite behaviors. Except through a direct logical implication. When a sub beginning implies the property. If that’s the case then that property is already useless. But in reverse too if no sub beginning can imply it then it is usable, it means something for the infinites.
Luckily to exclude these sub beginning implied, useless properties is very simple.
The implication means that all continuations possess the property while the non implication means that some will not. So, we have to require that the property should be avoidable from any beginning. That is, the negative of the property is continuable from any beginning. In short, the property is avoidable or the negated is continuable.
For example, to start with 0 is a non avoidable property because beginnings starting with 0 can not be continued to avoid the property. Which is bad because then those random sequences that start with 0 will obey this property for ever but clearly not due to their randomness. Also, the segment considerations will not be able to derive this property to continue and so we would have to judge it to be strange defying randomness. Yet it doesn’t really defy randomness, merely restricts it by some beginnings.
Now that we know this “practical” fine print, we still have to digress because there is an other even more important one. Indeed, to claim that the infinitely occurring, that is continuing avoidable properties in the random sequences are only the ones that follow by segment considerations can not be stated yet precisely, because the segment considerations are not usable for individual random sequences.
The consequences of these segment considerations by the way, will be called expectabilities, and so we have to find the framework for this. This will give also a new insight into the physical meanings of randomness. So, to separate these from the math, we shall use the word random in such physical sense. Only the final definition or interpretation of the final physical randomness should refer to randomness.

A beginning property is trivially occurring if in every sequence there is beginning obeying the property. To have fifty digits is such.

A beginning property is randomly occurring if in every random sequence there is beginning obeying the property. To contain fifty 0-s is such. Indeed, the random sequence will have infinite many 0-s and we just have to go up to the first fifty.

A beginning property is expectably occurring if in any sequence where our segment axioms are true there is beginning obeying the property. The previous, having fifty 0-s is this kind too because we can see first that having at least fifty 0-s is expectable namely by the Law Of Big Numbers using fifty long windows. Then the exactly fifty 0 is consequence of this by simply going up to fifty again.

A beginning property is trivially continuable if every beginning has some possible continuation that the new longer beginning obeys the property.
To contain unequal 0-s and 1-s is such because this is following from having odd many digits and this obviously achievable from any beginning.

A beginning property is randomly continuable if every beginning and every random sequence combined will have a beginning obeying the property.
Having equal many 0-s and 1-s is such. Indeed, we can count the 0-s and 1-s in the beginning, calculate their difference and in every random continuation we can go until the opposite difference appears in the continuation.

A beginning property is expectably continuable if every beginning and every sequence continuation of it for which our segment axioms are true there is a beginning that obeys the property. Again the previous equalization is such by the same logic.

A trivial axiom of physical randomness is that a random sequence is random from any point too. Thus, randomly continable property will actually infinitely occur that is be randomly continuing too in any random sequence. In fact, the reverse is true too:
Random continuability follows from the property not just occurring randomly but occurring for arbitrary long beginnings that is simply continuing.

For the expectable sequences this same will hold as theorem, so again “able” and “ing” will be equivalent.

The non trivial axiom of physical randomness is a different kind of reversal of the implication from “able” to “ing”. Namely:

Only the randomly continuable avoidable beginning properties can randomly continue.

We can not translate this into mathematical meaning, rather identify the randomly continuable with expectably continuable. Then this axiom becomes a definition that the random sequences are those where only the expectably continuable ones are the continuing avoidable beginning properties.
Or negatively, the sequences that have a continuing avoidable beginning property that is not expectably continuable, are strange. These avoidable beginning properties that are not expectably continuable are the strangenesses.

Observe that this whole idea is born from the failure to grab the random continuability directly as expectability.
The real physical random sequences are randomly continuing by themselves.
Our expectable continuabilities will be true if we regard sequences obeying the segment axioms. But we could not just say that the random sequences are these.
Because we can insert non random features even into sequences that obey all these.
Nature excludes these as our above non trivial physical axiom claims.
To exclude these mathematically, is the requirement that only the expectably continuable properties are continuing.
Thus we not only grabbed randomness but peeked into its physical mechanics too.
The root of randomness is the random continuability, that every beginning if continued randomly will climb into certain properties. Infinite occurrence, that is continuing in sequences is merely a logically alternative form. Probably when generalized to bigger sets and well orderings, will this nuance be appreciated.

But even now, the vision of this continuability is a wide but finite feature. Being so wide that is applicable to all beginnings, it translates to individual infinities.
Then a second point becomes that the other reversal is true in nature.
Nature only continues by continuabilities, it doesn't know the strange continuings.
In math, these strangenesses are actually the more important. And so Physics has to deal with it too.

The old view, the stopping of the random sequences in the machine generated and critically narrow, that is finite totaled beginning sets, focused even more on strangeness. It only wanted to exclude these. It offered only a special continuing in the random sequences. Namely, by looking at the negated property or complement set.
As we mentioned, this not only will infinitely occur, that is continue but will actually stay for ever from a point on.
For our example of self repeating beginning, this negated property is a beginning not being an exact self repeat. Obviously, the staying is the whole point here.
The much weaker claim of continuing, that is that there will be infinite many non self repeating beginnings, is so trivial that we almost ignore it!
Well, we shouldn't! Because as trivial as it is, it is again a case for my new point!
It again can be derived without the old randomness definition and complementation, rather from elementary segment considerations. Indeed, our last example, the property of having a new repeat champion, obviously implies that the beginning can not be self repeating and for that we already showed that it occurs infinitely.
In fact, that was again easy as consequence of being expectably continuable!
Indeed, any beginning has already a maximal consecutiveness in it and we simply have to find a first segment with more.

The old-fashioned thinking would make us believe that these expectable infinities, should tell the random sequences. Of course, a few being true is obviously not enough. Then we could think that requiring all of these to be true is the answer. But we are wrong again! All such derivable infinities is not only not enough but avoids the crucial new leap! To require not that all these are true rather that only these are true!

The weird non concrete consequences of the dreaded s arbitrary random sequence through Set Theory will not lead to contradictions now.
We welcome the existence and usage of such random sequences.
But now the definition includes the segment derivabilities of our system in a new way. Not as particular cases, rather as their whole set. Or for the strangenesses even more evidently as non derivabilities. This is a first in mathematics that a field defines a natural concept by its own derivabilities.

But we didn't exactly tell what to use as "segment axioms".
I propose the Law Of Big Numbers and the Law Of Equalization.
The first says that in a random sequence from any point, every fix length segment will come about in that long windows.
The second says that in a random sequence from any point, already a first long enough segment will have exactly equal 0-a and 1-s.
If I am wrong and these two axioms are not enough then someone will have to be able to show some beginning property that is randomly continuable but not obtainable from these two. So then we'll need more segment axioms.

A bigger problem would be to show infinite occurrence of a property in random sequences that is not a randomly continuable at all. Then the mentioned physical axiom of random continuability is not true.

The point to be seen is that this idea is an already existing reality formalized.
We already have the simple segment methods leading to random properties.
We merely ignore the system of these as a defining force simply because we are thinking in the old derivability cycle.

Both the old and the new randomness definition say that a sequence is random if it has no strangeness. Both even agree that a strangeness is something continuing that shouldn't. For both, this means that such strangeness then must stop in random sequences. The crucial difference is the meaning of the "shouldn't".
In the old definition it means that the beginning property or collection has so diminishing chances that the totals from later points are diminishing too. Thus the infinite occurrence has zero chance.
In the new definition the shouldn't simply means really that it should not! Or rather it "not should"! The should is the expectability. That the continuability is derivable from segment axioms. The deeper hidden assumption is that to go randomly from an arbitrary beginning and end up in the property is simply caused by that it is derivable.
The mysterious randomness is actually the human derivability!
This new primal randomness definition is actually quite testable.
Indeed, for a suspected randomly continuable property we "simply" have to pick all possible beginnings and start all possible random trials. Our claim is that exactly those properties will become true that are derivably continuable. The amazing biggest step of the new randomness definition is that we captured this primal randomness. This has nothing to do with random sequences. It 's about random continuations. This will become part of the new Set Theory already.

All the trivial strangenesses that we could tell at once subjectively as defying randomness, are now only strange if they imply some avoidable property that is not derivably continuable from segment considerations.
A most obvious old strangeness could be for example to have all 0-s.
This is easy as beginning property, having again all 0-s.
It is avoidable, because even the all 0 beginnings continued with something that contains 1 will avoid it. The others can be continued arbitrarily. On the other hand, this property is not continuable. Indeed, any beginning that contains 1 is uncontinuable into the property. This at once proves that the property can not be expectably continuable either. So, indeed, we have a strangeness in the new sense too!
For more complicated, continuable old strangenesses, it could be very hard to prove that they remain strangeness in the new sense too, because here we have to show underivability from the segment considerations.
So, the really important cases are the "free" properties that are both avoidable and continuable, that is both the property and its negation are continuable.
The concrete examples of free properties are all either expectably continuable for both the property and its negation, or are stop stay pairs.
For example, having more 0-s in a beginning, or its negation, having more 1-s, are both expectably continuable. And they indeed alternate in the random sequences.
For the other kind of example, we can start with the old naïve strangeness to have all 0-s from a point. This implies the continuation of the free beginning property to have all 0-s in the second half of a beginning. And indeed, this is not randomly continuing,

in fact, it stops in random sequences. Which we only know from the old randomness concept because the chance total is 1. This leads to a circumstantial derivation of the non expectability of the continuing. And thus also of the non expectability of a continuability. A pure derivation will be in a new Random Set Theory.
I can not give examples of free property pairs where none of them are expectably continuable or where only one is but it is not staying only continuing.
The first kind shouldn't exist because for any sequence and any negated property pairs only one of them can stop, so at least one has to continue. Then in a random sequence the same happens for any free pairs too and so the continuing one must be randomly continuing. Thus if my proposal is good then this should be expectably continuable.
So, the most likely death blow to my new randomness definition is to find a free pair of properties that neither of them are expectably continuable. Indeed, then all sequences are instantly strange, there are no random ones. But this death blow will actually be a start of taking the new definition seriously! Just as lambda calculus started with a contradiction. Here, this will lead to restrictions of the properties and maybe new segment axioms. Then proving that one of a free pair is always expectably continuable, will be the first fundamental theorem of the new Random Set Theory.
The other kind, alternating property pairs with only one of them expectably continuable, while the other being strange, must exist and will hide the tricky new strangenesses.
Non beginning properties of course can be true for some random sequences and we already mentioned being an omega number as an example. There will be many others and so the infinite distinctions between random sequences will cease to be a "strangeness" subjectively. In fact, these will decide whether the Continuum Hypothesis is meant to be undecidable or will have some further restrictions beside the trivial one we know today.

These last few pages I had to change at least a dozen times. The previous twenty pages came out from me under one night after I realized that I wasn't happy with the four page article "Expectability", that I mentioned in the beginning.
My struggle with the end pages, to tell clearer the new discovery, or rather the remembered old message, went on daily in the last two weeks.
The basic thought remained the same but the minute conditions became precise gradually. So, these sentences are not in the copy that I sent to Girard and the American Mathematical Monthly.

A final thought:
Randomness is not the first physical concept that mathematics struggles and actually can not grasp perfectly.
We had already a clear cut concept that was physical and clearly non mathematical, seen by mathematics itself. Namely, the concept of Effectivity. Then all the misleading hocus pocus with computers covered this simple fact. Some parrots even dedicated their lives to rename all definitions to "computabilities". The already existing idiotic names like recursive enumerability for the most fundamental effectivity, helped this insane computability renaming but actually the new stupidity conserved the old as well. The real point is that real computers are not really effectivities mathematically and even create false plausibilities.
Of course, the new computer age did become a reality and even offered a new army of mathematically oriented students. But lets not fool ourselves that they have an instant passport to mathematics.
Unfortunately, the ignorance happened in much higher circles too.

Einstein walked daily the park of Princeton with Gödel. They probably did not discuss their thoughts in their fields (quite absurdly I might add), but Einstein had to know about Gödel's old results. And yet, he hoped for the "best", that he could be lucky again and so his intuitions can fish out the right mathematical concepts. But he was fishing in a wrong pod, namely old mathematics. Unifying Relativity with Quantum Mechanics needs a much bigger fish.
So my naivety when I was sixteen and sent letters to Nobel Prize winner physicists, and Robinson's prologue to them are too early attempts toward the unavoidable:
The points can not be the elements of space and time. Zeno and Parmenides were right and Euclid "missed" the "simple" concept of point sets as geometrical objects also for some deeper reason. The amazingly late realization of the Fractals is a reawakening of the deeper problems with our continuity intuitions.
So, continuity is a third and probably most fundamental physicality that is not capturable mathematically by simple axiomatization.
Random sets will be a new connection to future physics and a new continuity too.