Expectability

A New Definition Of Randomness

A New Future For Set Theory

Forty seven years ago, still in high school, I created a naïve definition of Randomness that was practically identical with Solovay randomness.
I spoke not a word English, never heard of Turing or any other form of Effectivity.
I regarded the B beginning set indeed just naïvely as a list and the finite total of the chances as a fast diminishing of the length groups.
First I regarded a beginning property that has the n length as variable and I realized that the p(n) chance function of this property relates to whether we feel an infinite repetition of the beginning property plausible or not.
I was running around to all my family members and asked them the following:
Do you see anything special here: 0100101001001010 . . . ?
Of course they didn't and then I drew their attention to the fact that the first three, then the first five and then again the first eight digits repeat. This was only important to get their attention and then I asked the real question:

Is it possible in a random sequence to find infinite many such repeating beginnings?

To my greatest satisfaction everybody got it right and said that it would be impossible. But this pleasure turned to pain because I also realized that this perfect intuition is a muddy water. Namely, the feeling that these beginnings should stop was mostly caused by the incredible fact that the first half that is supposed to repeat is an arbitrary history. So, the intuition was not caused by the true reason, which is that the full repeated double beginnings have some very small chance values. This missing essence from the intuition becomes obvious if we ask to regard not the beginnings, rather increasing segments in a sequence. So the first outcome, then the next two, then the next three and so on. These are independent trials and the chance consideration implies that among the even long ones where repeat could be, again only finite will come true. The doubled segments will stop. Yet, very few people will feel this as a necessity.
Regarding longer and longer even segments, the chance of repeating ones is getting smaller and smaller. For two lengths we have half of them being repeating 00 and 11 but at four length we only have 0000 , 0101 , 1010 , 1111 that are only a quarter of the total sixteen possibilities. This always halving diminishing is obviously not sensed by our intuitions and especially the true feature behind it. Some sense of the diminishing, could be involved in our a priori intuitions but the distinctions among diminishings would be beyond even my idealism. Especially, if we realize the correct distinctions. It's clear that an always proportional, that is inverse exponential decrease is fast. In fact, the media uses this "exponential growth" expression without any specific meaning, merely as a substitute for fast. But these learnt expressions have nothing to do with real intuitions.
I was only fifteen but soon figured out that the point that makes the decrease "fast" and thus the infinite repeats impossible, is not the exponentiality or any other functional speed, rather that their total is finite. This of course involved the recognition that values could diminish and their total still can be infinite. I called this the Anti Achilles paradox and instead of the usual start from the reciprocal sum, I was

able to show to a naïve eye that infinite many equal distances cut into more and more pieces are a trivial visual example of this fact.
It was then easy to realize too that the even more important point behind the finite sum was a most primordial form of the zero one law. A black and whitening of the situations. Namely, the total sum not from the start but from later and later points to infinity must either diminish or stay infinite.
So, the finite total simply means diminishing of the rest after any value.
Finally, this lead me to realize that this diminishing of the rest of the chances in sum can be translated into a product of the complement chances. So I figured out slowly the Borel Cantelli Lemmas.
My main concern remained the randomness for a few months and first I formulated the law that the chance total of the p(n) values must be finite to avoid infinite repetitions of the property in a random sequence. Then I changed this to merely regarding a B "pool" of all usable beginnings and a sequence has to be built from this pool. A random sequence simply can not be built from a finite totalled B pool.
Or to put it more visually, it stops in it, it can not go through The simplest exact claim of course is that a random sequence can have only finite many beginnings from B.
An other vision is that a random sequence can not have arbitrary long beginning from a given finite totalled B. This is especially visual oppositely, that a non random or "strange" sequence must have arbitrary long beginnings from some B finite totalled pool. So these finite totalled B-s are actually the strangenesses.
I also realized that many non fast diminishing beginning properties, that is where the total of B is infinite, can be shown to repeat infinitely by simple arguments about segments. I called the Law Of Big Numbers the simple fact that whatever has a chance, must come about if we try it a big enough many times. Thus of course it will occur infinitely often if we try it in a sequence. I emphasized that while this is indeed a law of plausibility, the Law Of Large Numbers is a deception. It seems to claim an infinity as randomness but in truth it is a stopping, a finiteness as it should be by the recognized randomness definition. Anyway, my simplest original first case where such application of the Law Of Big Numbers gives the random infinite repetition, was the following beginning property:

The last digit of the b beginning repeats consecutively at the end more times than anywhere earlier. Like 01001011100010011111. The 1 at the end repeated five times and before the most was three.

For longer and longer beginnings this property is having less and less chance but still must happen infinitely. Indeed, we must have arbitrary long full 1 segments and the firsts of these with the beginning before, are always such beginnings.
That we must have arbitrary long full 1-s is simply visible if we regard arbitrary long consecutive windows. In these windows all possible combinations must come about starting from any point. So actually infinite many times.
I even realized that this simple argument avoided the whole non fast diminishing just as the original repeating beginnings intuitive stopping avoided the fast diminishing.
But the Law Of Big Numbers does not tell something important about segments.
It visualizes the fix long segment as tried simultaneously under each other and thus creates every possible outcome combination. This of course can be also visualized as trials after each other as windows. But we can also regard arbitrary long segments simultaneously and then the "big" claim is that for a long enough segment already the first trial gives equal amount of 0-s and 1-s. With windows in an infinite sequence this means that going far enough from any point, an equalization will happen in a

coming long enough segment. This of course means infinite many equalization windows and this also means the same for the whole beginnings. Finally, combined with the Law Of Big Numbers this easily implies arbitrary big oscillations in the 0-s and 1-s from the beginnings too. We just have to imagine new windows after the equalizations and find full 0-s or 1-s there.

Now comes the spooky part!

The crucial idea that these infinitely repeating beginnings in the random sequences should all be obtainable from a few principles like the Law Of Big Numbers and Equalization, through conventional mathematics, completely disappeared from my memory for almost five decades. Only few days ago in a dream did I realize this again. And when I woke up it was very hard to reconstruct the long forgotten idea. As I said I didn't formally knew about effectivity and yet I knew that this definition would mean something revolutional. Indeed, by this definition we could nicely obtain the random continuing beginning properties but to claim a random sequence would mean to prove that all of its continuing beginning properties are derivable. What is even worse is that the non random, that is strange sequences that we can normally recognize by some particular and usually obvious single strangeness, become an even more complicated affair. Indeed, now we have to show that this particular continuing beginning is not obtainable from the segment principles. With hindsight of course this means that these non obtainabilities could be themselves derived by a secondary system. If for example we can prove that all such and such beginning properties or beginning pools automatically mean non derivabilities then we can obtain individual strange sequences again easily. But this means a whole new game.

In math we never had such natural phenomenon that would have involved the considerations of derivabilities themselves.

Obviously these wild ideas that came from nowhere and the just learnt conventional math that was itself new to me conflicted and that's why I not only abandoned but actually suppressed the whole idea.

There were two other "wild" ideas obtained by me from nowhere.

One was forcing and the other non standard analysis.

I even wrote a letter to Werner Heisenberg to suggest the new reals as a possible foundation for a wider physics. You can imagine my astonishment when I read years later Robinson's prologue to physicists. The other mystical visions of forcing I was lucky to tell in person to Paul Cohen when I met him.

The really strange thing is that these clearly supernatural insights in high school did not make me an idealist and neither did I realize that I was merely a medium. I felt special. My ego was still deluding me.

To share these in a world where all personal revealings are regarded as signs of lunacy is a risk that I have to take. I already know that this world is crazy and not in a funny or cynical sense that would mean nothing more than a new layer of lie, rather in a tragic and emotional sense. And indeed, to know must involve emotions.

Returning to our point, my rediscovered old idea is very simple:

Instead of striving for the strangenesses as I did originally and everybody else since then too, we should aim directly to the randomly infinitely repeating beginnings!

These are all and only those infinities that are obtainable from the Law Of Big Numbers and the Law Of Equalization!

To derive these infinities precisely we have Set Theory with the added new concept of a random sequence and the crucial new axioms of the Law Of Big Numbers and Equalization.

Since these two laws are telling expectable random occurrences, we should call this approach as the “expectability” definition of randomness:

A sequence is expectably random or simply expectable, if every infinitely occurring beginning property in it is derivable from the Law Of Big Numbers and Equalization through segments.

I add three important points that are relating to the older randomness concepts:

The infinite chance totality of a B beginning pool does not imply the infinite continuability of the random sequences from B by using the second Borel Cantelli Lemma and regarding 1 chance as certainty, because the beginnings are not independent! The finite totalled B pools as strangenesses was more implied by the first Borel Cantelli Lemma but it still wasn’t a proof either.
In spite of these strange facts, the Solovay or Martin Löf or Chaitin randomness is a very good approximation of this expectable randomness, that we simply ignored up until now. We can ask why and how far this approximation works.

To use Set Theory was already suggested by many but in a much more complicated manner. Namely, by allowing all tail properties about sequences and then split those as the regularities and irregularities. Returning to simple beginnings is much cleaner and more promising too. Namely, it could rejuvenate Set Theory itself!!!
Not just implanting the random binary sequences into Set Theory and thus stepping toward explaining why the Continuum Hypothesis has to be independent, but also generalizing randomness from sequences to wider structures.

Finally, the third point relates to the seemingly also directly random interpretation of the Martin Löf version. This was created of course before the Solovay recognition of the finite total and infinite occurrence as “obeying”. In fact, only later was the single pool of a B used by Downey. But going in reverse in time or going from my naïve original idea, the Martin Löf sequence of beginning sets are a narrowing of B. The infinite occurrence is replaced by occurrence from every element of the narrowing sequence and the finite total is replaced by the requirement that the partitions are diminishing by some mechanical manner too. This most important element was totally hidden in Löf’s original definition by simply using the fix half, quarter and so on coverabilities of the sets. These values suggest some kind of probabilistic meaning for the total. But actually the reciprocals could be used too. And these are slow diminishing. But it doesn’t matter because any chosen subset of these narrowings can work as new narrowing. The knowing of the reciprocalness was important though! This knowledge allows the selection of a subset from the narrowing chain. Of course, such formula given narrowing still hides the real point, the mechanically diminishing, which we should call “hard diminishing”. This concept overrides the distinction of slow or fast diminishing. We can always choose fast from a hard too.
So, Martin Löf’s randomness actually says that:
Narrowing beginning sets that are given by machine and also diminish hard, will determine sequences that can not be random. These sequences are the strangenesses.
But amazingly, there is an alternative view that the narrowings are randomness tests and the failures in them, the not going through all the way, rather stopping at a narrowing set, is showing the “level” of randomness. This is the fully accepted view and Löf himself started with statistical tests. And yet, I disagree with this view.

The infinite statistical properties of random sequences are very distorted in these Löf narrowing sequences of beginnings. They are simply strangenesses as whole, that is when they all give the continuing beginnings for a sequence.
There is an other even more widespread false meaning projected into this narrowing of the single B pool. Namely, the always repeated point is the measure meaning or the catchy phrase "effective nil cover". The infinite sequences are viewed as points and everything seems to become geometrical. The already mentioned usage of half, quarter coverable narrowings in this simplified version is even worse because the "effective" seems to suggest that the sets are given effectively. The real point is that the narrowing is given effectively that is hard.
The fact that this is the point, comes out from the real advantage of the narrowing of the single B. Namely, that we can alter the whole definition in a crucial manner.
Strangely, exactly by dropping the very thing, the hard requirement. So, just regarding machine given but arbitrarily narrowing beginning sets. This weakening should mean more possible sequences becoming strange! So less declared as random! Yet not any intuitively random sequence becomes strange. So what's going on? As it turned out, the famous Chaitin omega numbers are excluded by this dropping of the hard requirement. Probably Chaitin was the only person who so strongly believed in his algorithmic randomness that his explicitly definable omega numbers being random didn't bother him at all. Most naïve minds feel that concretely given random numbers are just not okay. But actually, this is unavoidable! Here of course, for the omega numbers, the concreteness not only means explicitly by a formula but determined by a machine. So, many used this extra fact as a false argument to improve randomness by going from the hard covers to arbitrary. These demagogues didn't realize that this step doesn't fix up the real problem that concrete random sequences will remain. In fact, since they just stepped from the particular half, quarter covers to the general diminishing, it gave the false impression of actual generalization there. The real point of the Löf randomness is exactly the hard diminishing cover. It brings out this hidden feature in the Solovay's and my old naïve definition. The hard diminishing will become a standard feature in the new non standard method of approaching the expectably random sequences.

To show that I am wrong and this expectable randomness is a false idea, one would have to give a concrete beginning property that is continuing in random sequences, yet is provably not provable by the Law Of Big Numbers and Equality.
So here is the challenge! Prove me wrong! I think that the person who will give a counter example, will also believe in this approach so much, to merely refine it and add extra other Laws to these two. The real point is that this road exists:

An arsenal of infinite beginning occurrences in the random sequences are obtainable by basic segment principles and basic set theoretical methods.
This implies the necessity to use this as a definition of randomness.
This then means a first case in mathematics where a natural concept is defined by the system's derivabilities and non derivabilities themselves.
Then this means a road to undecidabilities to have internal meanings!
This is a revolution of the axiomatic method in general and Set Theory in particular.