Fundamental Physics

Cycles Of The Moon

The brightest object in the night sky is the moon, in fact it is so bright that it can illuminate the earth and we can actually see in a forest. The trees will even cast shadows, so “moon shadow” really exists.
The most obvious change in the night sky is also happening to the moon, namely it increases and decreases its size, day by day, in a 28 day cycle. In other words every 28 days we’ll see full moon, while in between, it gradually becomes smaller and after 14 days it almost disappears and then again increases to full moon. This moon cycle was believed to cause some biological effects, most well known as the cycles of menstruation. Even depression is believed to coincide with the full moons.
A more basic question was what causes the change of shape at all? Looking at the crest shaped moon, even with our naked eye we can see sometimes the other missing part. So it became obvious that it is some light trick that causes the apparent shapes. If we look around in our own surrounding, we’ll soon discover that similar crest shapes appear if we throw light on a ball. A big difference is that on earth, the air leads some light to the shades, so they are not as dark. If the moon is a big ball without air on it, then the sharp crest shapes are quite natural as the halves lit by some light source.
The next obvious question is, what throws light to the moon? The amazingly simple answer is, the sun! This of course means, that when we look at the moon, lit in the night sky, then that light must come from the sun, being behind us that is, above the other side of the earth. And indeed, on the other side of the earth, it is daytime, when we are at night.

Man On The Moon

All this was figured out by people in ancient times already, but it still remained a question how big the moon actually is. The assumption that it is huge body like earth came much later.
Today, we’ve been to the moon and verified exactly all earlier theories.
It is one-quarter in diameter of the earth, without air. This not only means that the shades are much sharper and darker, but that the differences of the sun’s heat is even more drastic. Since, there is no air that filters the sun and distributes it’s heat, it will be boiling hot where the sun is lit, while freezing cold in a shade, just a few centimeters away.
Surface gravitation, that is weight of same objects on the moon is one sixth of earth’s.

The Daily Turning

Every night we see the moon and the stars slowly turning from east to west just as at daytime the sun comes up and goes down. People even realized that this “turning around” could be simply an illusion, if in reality the earth is spinning and makes one full turn every 24 hours. In daytime we are turning in front of the sun, while at night we continue the turn with the sun behind us and thus, seeing the stars and the moon.

The Wandering Of The Moon

Watching the night sky for one night, we only see the moon turning together with the stars, but if we keep on watching it day by day, we'll notice that the moon's position is changing and after a few weeks. It will be among totally different stars.
The 28 days cycle of the moon phases might suggest that not only the full moon, but the position among the stars will return in 28 days. This doesn't happen because the 28 days is not exactly 28 days! This is understandable if we realize that this cycle is caused by the orbiting of the moon around the earth. And indeed, why would the orbiting time of the moon be an exact multiple of the spinning time of the earth itself.

From Wandering Stars To Planets

The phases and the wandering of the moon are the most striking appearances for any observer of the night sky. Amazingly, a handful of yellowish stars will also wander around when observed week after week. These wanderings are even more unpredictable and are not related to each other either. A logical explanation could be that the normal stars are very far from our sun, earth and moon, while these wandering stars are moving somewhere nearby by some rules but appear wandering looking from our earth. From this, then it is really just one step to realize that the normal stars are suns giving their own light, while the wandering stars are objects reflecting our sun's light, just like the moon does.
This of course, became only a certainty when we started to use telescopes and looking at the wandering stars, they appeared as discs, rather than points. The name planets were then given to them, which comes from the word "plane", that is having a surface. This has nothing to do with the plane in which the planets might move!

Yearly Cycle With Fix Tilt

Beside the daily spinning of the earth, the most obvious cycle is the full year.
This contains the change of seasons. The cause of this is the following:
The incoming sunrays to a fix point of the earth's surface, will sweep a half plane from 0 angle at dawn to 180 at sunset every day. But this plane is not perpendicular to the surface of the earth, so for example at noon the 90 degree angle sunray is not 90 degree that is perpendicular to the earth surface. In short the sun is not perfectly above us even at noon. This tiltedness of the daily half circle of the sun rays is changing slowly day by day. In summer it is smallest so the noons are highest, while in winter it is flattest, the noons are lowest. Of course, lower angles of the sun rays mean less heating power. So we get continuous changes of temperatures too.
But what causes this alternating tilting of the sun rays? We might think that it has to be caused by a new yearly period in the directions of the sun rays, that is in the directions of the connecting lines between the sun and the earth.
If we accept that the earth is spinning and this gives the false impression of the sun circling us, then the simplest geometrical explanation of the year could be that the sun is going up and down every year and this creates the changing angles.
If we follow the bible and the church and claim that the earth is fix without any motion, then strangely we get a simpler explanation. Indeed, then the sun has to circle us daily already and so simply these circles are slowly changing from higher to lower ones and back every year.
Of course the sun centered view started to reappear after the planets were realized.

But amazingly, the seasons also fit into this world view much better if we realize a crucial missing element that we ignored about the seasons. This is that the northern and southern hemisphere of our earth have opposite seasons.
So, it is not the sun and earth connecting line as simple sun ray line that is changing, rather the particular incoming angle on the surface of the earth.
Could the earth's spinning axis, the north and south pole connecting line, make some yearly alternating tilting and thus expose opposite hemispheres to bigger angles?
It could, but there is a much simpler explanation. Namely, it's enough to have a fix tilt of the earth axis relative to the plane in which the earth is orbiting the sun every year. Indeed, in summer of the north hemisphere, the north pole is tilted toward the sun. After half year, the earth is on the other side of its orbit but its axis is the same. This exactly means that now the south pole is closer to the sun.

Planets Around The Sun

The earth's orbiting around the sun almost dictates the obvious consequence that the planets do similarly, or in other words, we are merely one of the planets of the sun. Then, these circling other planets could indeed, be appearing from our planet as meaningless wandering loops. It was also realized that closer planets must orbit faster, while further from the sun must go slower and thus, we could even map out where the planets must be. Mercury is the closest orbiting fastest, that is having the shortest year. Then comes Venus, then Earth, and further out Mars. These four are also called the inner or rocky planets. A whole belt of asteroids and minute planets separate these from the four giant or gaseous planets: Jupiter, Saturn, Uranus and Neptune. And finally a small, distant, rocky Pluto ends the solar system.

Invisible Stars

Where are the stars when it's daytime? Only few children ask this question and only few students who are taught about the solar system. When they learn the truth they are amazed. Indeed, it's amazing that the stars are all over behind the blue sky and we simply can't see them because we are blinded by the sun's strong light. But the most amazing is how simple is to prove this. All we need is a long tube attached to a little ball where we can sit inside in total darkness. When we look out through the tube, the sunlight will not get inside and if somebody positions our little capsule in the right directions, we can see all the stars. Similarly, if we go down into a deep well or inside a tall factory chimney, looking up we could see some stars if they are pointing in the right direction.

Eclipse Of The Sun

There is an other way to see the stars at daytime, but we have to wait for the right time. Sometimes the moon that orbits the earth every 28 days, gets exactly on the distance that connects the earth with the sun. From the earth, amazingly, the moon is just about the same size as the sun and thus, if the alignment is perfect, the moon will cover the sun for a few minutes. The Egyptians were frightened of these eclipses, but their priests knew pretty well what's happening and observed the stars, that appeared for the few minutes. An amazing application of this ecliptic daytime star appearance was achieved in the 20-th century to verify Einstein's Theory of Relativity. They looked at two stars that appeared on two sides of the eclipsed sun. Measured their angles and compared it with the angles that can be measured normally at nighttime, when the sun is not in between the two light rays. As we would expect, if light contains matter particles, then the sun's gravitation would attract these a bit and even though it couldn't capture them, it would bend them a little bit. So, the ecliptic angle should be bigger than the nighttime angle. This has nothing to do with relativity yet, and follows from Newton's gravitation too. Relativity changed Newton's Laws, but only considerably for very high speed objects like the light particles. So it predicted much bigger angle change than the Newton Laws would require. The experiments perfectly confirmed the bigger relativistic angle increase.

Eclipse Of The Moon

If the moon crosses the connecting line of the sun and the earth, but not in between them on the distance, rather on the other side of the earth, then this means that the sun won't be able to lit the moon for the few minutes that the earth passes through. In other words, the moon will become dark for a few minutes at nigh. An eclipse of the moon will appear. So, instead of the self shade of the moon that we see every night as its shape, an actual shade of the earth will cover it for a while. This is not as scary as the eclipse of the sun, but just as surprising.

Galaxies

Today we know that the stars or suns themselves form big groups that can have different shapes and rotate around a center. These are called galaxies and our sun is in the milky way galaxy. Most of the "stars" that we see are actually whole galaxies so far away, that they melt into one point. Unfortunately, we don't see the depth in the night sky, so "nearby" suns of our own galaxy look the same as far away galaxies. With telescopes, we can see the magic of how a single "star" can become a whole cloud of stars as the magnification is increased.

Olber's Paradox Of The Infinite Depth

If the universe is infinite and filled in with galaxies, then looking at the night sky we should see just as much light as daytime. Indeed, in an infinite space looking at any direction, sooner or later, we should encounter a galaxy. We could argue that it might be too far to send its light, but if we think about it further, it turns out that the weakening of the light should be much more compensated by the other possible sources around the same direction. Best way to feel this is to imagine an infinite grid of fix cubes having a light bulb in every corner. No matter how big the cube is, the light bulbs would blind us in every direction. The obvious two ways to avoid this light paradox of Olber, is to conclude that either the universe is not infinite or, that the galaxies are not filling the universe regularly, they become rarer and rarer, further away from us. This would mean that somehow we are the center of the universe. That's what the church believed too, when insisted upon the earth being the unmoving center of the universe.

Tycho Brahe's Book Of Facts

As we said, the ancients already realized the wanderings of the planets. They plotted the planet's path between the fix stars. A much more accurate system would be to write down the positions of the planets measured by a giant protractor. Basically, all we need is a big tube attached to a turnable arm which itself is sitting on a plate that can also turn around. Tycho Brahe constructed exactly such instrument and spent nights after nights writing down the positions of the planets at every repeating interval, say ten minutes. His data filled a huge book that was only known by few of his associates. One of them was the young Kepler. Brahe heard about the sun centered explanation of the planets, but when he made the calculations they didn't match with his observed positions.

Kepler's Ellipses

As we said, it was quite obvious that planets closer to the sun should move faster, that is have shorter years. Indeed, if we attach a rock to a rope and spin it around, then pull in the rope, the orbiting will speed up. Kepler realized that if the individual planets themselves get closer and further from the sun, that is orbit, not around circles but on a little bit distorted or flattened loops, then they don't have to travel with the same speed all around. In fact it's natural that when they get closer to the sun, they speed up, while when they get further, they slow down. This whole thing might have just remained as a good idea, had not Kepler been an exceptional mathematician too. He soon realized that the flattened circles are in fact ellipses and that there is an easy way to calculate the slow downs and speed ups too. All we have to assume is that the connecting distance between the sun and a planet continually sweeps the same area under the same time. When the planet is closer to the sun, the connecting distance is shorter, so in order to sweep the same area, the planet must speed up, while when further away it will sweep the same area with a slower speed. Amazingly, when he made the detailed calculations, they all matched with Brahe's books, which he was able to get hold of. Besides these two laws of the elliptical orbits and fix swept areas, he also calculated the full orbit times for each ellipse, that is the years of the different planets. He even created a fourth law that would describe the positions and sizes of the different planets. This fourth law turned out to be a mistake later.

Galileo Under Pressure

Kepler knew Galileo, in fact sent him a letter, asking why doesn't he stand up for the sun centered view of the planets. Of course, while Kepler lived in a protestant country, Galileo was in the middle of catholicism. We all heard how Galileo made his experiments by dropping objects from the Pisa tower and invited the priests to witness the facts that contradict Aristotle. Aristotle made many scientific mistakes, the three most important physical ones were that concerned Galileo. The first two mistakes describes the falling of bodies, while the third claims the earth to be the center of the universe. Refuting the first two, would show that Aristotle could be wrong in the third too. Of course, the church maintained the earth centered universe, not only because of Aristotle, but mostly because of the bible. At any rate, when finally Galileo obtained a "good" telescope and saw the moons of Jupiter, this finally and completely convinced him that if Jupiter has moons just like the earth, then the earth could also be just a planet, like Jupiter. So he spoke out for his beliefs and was sentenced to house arrest by the church. Then he retracted his opinion, but told the famous saying to his friends: "The earth still moves".

Paradoxes Of Aristotle's Falling

Aristotle claimed that a dropped object falls with fix speed towards the earth, but different objects fall with different speeds, namely heavier objects fall with higher speed. He could have made similar experiments as Galileo did, but didn't bother, because trusted his common sense. Amazingly, even without experiments, just by simple logic, one can realize that his "common sense" was false. First about the fix speed, imagine that we drop a little pebble from one metre on the sand and then from much higher. Obviously, from a metre it only makes a small dent on the ground, while if we drop it from much higher, it will penetrate much deeper. If the pebble would fall with a fix speed then the fall of the last metre would be the same as the fall of just one single metre, so there shouldn't be a difference in the impact. If we drop the pebble on somebody's head, the difference is even more "drastic". The only explanation is that the pebble is increasing its speed and thus, when dropped from higher, will impact the sand with a higher speed. We are simply confused because our eyes can't follow the increasing speed. On the other hand, to refute the faster fall of heavier objects, we should imagine the followings: If we drop two identical objects like two dices, they obviously fall together. Now if we glue them together they still must fall together, even though they are twice as heavy now. But even if somebody would insist on that the gluing together could have a magical effect on the falling, we can still argue the following way: Lets glue a third one to them! By one logic, then the three together is even heavier so should fall faster than two. But, with an other logic the two wants to fall faster than the third, so this third one is slowing down the other two, so the three is slower than just the two. So the only possible reality is that regardless how heavy an object is, it will start from 0 speed and increase its speed always the same way. In other words, the achieved speed only depends on how long the object has been falling.

Galileo's Odds

The main difference between Galileo and Kepler was that while Kepler was a good juggler of mathematics for his physical intuitions, which became a typical feature of all successful future physicists, Galileo was an old fashioned slow user of mathematics, always going deeper and deeper and thus unable to hit upon the best mathematical form.

The falling together or the "Law of Common Fall" is pretty straight forward to demonstrate. All we have to do is drop a ball of lead and cork together. Of course, lighter objects sometimes really do fall much slower, like a feather that glides slowly. But these are caused by the air. Even a cork ball will stay a little bit behind because the air slows it down more than the lead ball.

The increasing speeds of fallings is more tricky to show, after all we have to measure the falls under certain short times. And that's where Galileo made a bad choice of mathematical formalism. He measured the falls in consecutive seconds and realized that if the fall in the first second is d_1 distance, then in the second second it will be $3d_1$, in the third $5d_1$, and so on, in the n-th second the fall is $(2n-1)\ d_1$.

He thought that the odd numbers are important here, which is a mistake!

It's a mathematical law that the sum of the odd numbers are actually the squares:

$1+3=4=2^2$, $1+3+5=9=3^2$, and so on, $1+3+\ldots+(2n-1)=n^2$.

Indeed, lets write our sum under itself, but in opposite order, member by member:

$$\begin{array}{ccccccccccccc} 1 & + & 3 & + & 5 & + & \ldots & + & (2n-3) & + & (2n-1) \\ (2n-1) & + & (2n-3) & + & (2n-5) & + & \ldots & + & 3 & + & 1 \end{array}$$

Adding up member by member the top and the bottom one, we get the same $2n$ value n times, so the two sums together give $n\,2n = 2n^2$. So one sequence is n^2.
This means that if we measure the falling distance, not in the consecutive seconds as Galileo did, but from the initial moment, then after t seconds, the drop is $t^2\,d_1$.
The beauty of this formula is that it can be used for not only whole seconds.
For example, after 2.6 seconds, the fall is $2.6^2\,d_1$.
Or after only half second, the fall is $\left(\frac{1}{2}\right)^2 d_1 = \frac{d_1}{4}$.

A New Common Sense Of The Common Fall

As we said, Aristotle accepted that heavier objects fall faster just by "common sense". We also showed why this "common sense" was faulty. Now we even claim that an other common sense completely justifies the Law of Common Fall.
All we have to do is to accept that the weight is a force that moves the falling body down. We might say that this idea is working exactly against us, because the weight and thus, the moving force is different for different bodies and thus, causes different motions. But not really! By accepting that the weight is a moving force, we have to first look at how already known forces would move a body. If for example, we try to push a big stone, with our force, we at once realize that it's not only our force that will count, but also how big the stone is. Obviously a heavier stone pushed with the same force, will only move less than a lighter one. In other words, the weight of a body gives an ability to resist any force that tries to move it. We can even realize that this resistance against forces is not really caused by the weight of the body, rather by its sheer mass. If we take our stone from the earth into the empty space, far from any galaxies, and try to push it there, it would still resist our force. So it's the mass of the body that resists any force and on earth this mass causes its weight too. We can even show this already on the earth by putting our big stone on a boat. Then pushing the boat, we really only work against the mass of the stone, because its weight is balanced by the water. We feel that it glides as easily as is possible, almost as in empty space.
So then, just as we have to conquer the resistance of a mass when we use a force, why would the force of weight be different. The force of weight is somehow caused by the earth, but this is immaterial, because that weight is still just a force that has to conquer the resistance of the body. So when we drop a stone, the mass of the stone creates the weight, which in fact is merely a force that tries to move the stone towards the earth. This force has to conquer the resistance of the stone just as if we would push it. And then of course, even though a heavier stone has a bigger force moving it downwards, this force has to deal with a proportionally bigger resistance too. So in the end, the bigger force and the bigger resistance cancel out each other and thus creating the same motion for all dropped bodies.

A Perfect Picture Of The Common Fall

In our previous common sense argument for the common fall there was still a very big hidden assumption, namely that the mass of a body is causing exactly proportionally its resistance against forces in general as the particular force, its weight, when it's nearby a planet, moon, sun, and so on. Or to go even deeper, we used the concept of mass as a wild card without giving any physical meaning for it, that would suggest the proportionality of resistance and weight.
Such meaning of the mass can be easily given if we assume that all matter in the universe is eventually built up from final and fundamental identical little units. If we

call such units as the God particle, then any object's mass could be simply regarded as the number of God particles in it. Then, it's quite obvious that the resistance of a body against a moving force is simply the resistance of one God particle multiplied by the number of God particles in the body. Also, the weight of a body is simply the weight of one God particle multiplied by their numbers. So, when the lead ball and the cork ball are falling, then simply two packets of God particles are falling obviously together. The lead ball merely contains more than the cork, but one single God particle would fall the same way.

Moving Force Specified

In our arguments earlier, we only stressed that the force that moves a body does not alone determines the motion, but the mass as the resistance of the body counts too. If we ask how the force and the resistance determine the motion, then the conditions on earth are not ideal to find the answer. Indeed, there are too many interfering circumstances. When a horse pulls a wagon, then the horse interacts with the road, the wagon also interacts with the road, the wagon itself has moving parts like the wheels, even the air slows down the motion.

It's much better if we imagine our heavy stone placed out in space and try to find out how it should move when an f force is applied to it for a t time. Obviously, it will start to move and finally reaches a v speed. It's quite natural that bigger force under shorter time or smaller force under a longer time, can achieve the same v speed, so $f \cdot t$ is that actually causes the v speed. If we imagine different stones to be moved with the same $f \cdot t$, then we also realize that the achieved speed will be different too. A heavier stone or to be exact in the empty space, a stone with a larger mass will only achieve a smaller speed, while a lighter one becomes faster with the same $f \cdot t$ applied to it. So, the required $f \cdot t$ is depending not just on v, but on m too, thus we obtain the fundamental $f\,t = m\,v$.

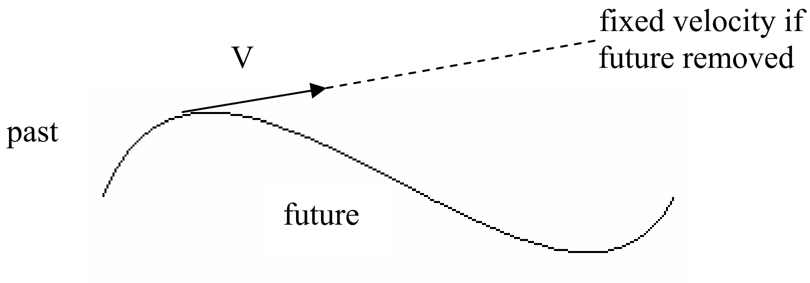
This equation can be further generalized if we realize that $f\,t$ can be applied to not only a standing stone, but to one that has an initial v_0 speed already. We might think that an already moving body is easier to speed up than starting from total stand still, but we are wrong. It's only the change of speed that counts, that is $f\,t = m\,(v - v_0)$. In fact, if instead of increasing the speed from v_0 to v, we use an opposite force to slow it down from a bigger v_0 to a smaller v, still $f\,t = m\,(v_0 - v)$ will be true.

Newton's First Law

In the $f\,t = m\,(v - v_0)$ equation, if f is 0, then obviously on the other side $v = v_0$ must be. In other words, the body will keep its original v_0 speed after t time too. In fact, if there is no force the body keeps on moving with the same speed forever. This is not only true for the size of the speed, but also for its direction. The directional speed is usually called the velocity. This could be best imagined as a so called "vector" which is an arrow, so that the size of the arrow is the size of the speed, the line of the arrow is the line of motion and finally the direction of the arrow shows in what direction the body moves on the line.

Changing from numbers to vectors is the crucial step that Newton took to formulate the laws of mechanics. To use the $f = 0$ case as a special law was important, because this way we could avoid the complications of f itself, being a vector and thus only concentrate on the vector feature of v. From now on, we'll use capital letters for vectors, so the velocity should be denoted as V.

If we only care about the length of a vector, then we use the absolute value $|V|$.
So for example for the velocity this means the old fashioned speed.
Claiming that a forceless body keeps its velocity forever is important for two other reasons too.
Firstly, it gives a physical definition of the velocity itself. Imagine that a body in non forceless, that is realistic environment moves in some path. The moment by moment velocity of the body could be mathematically defined from the displacements and the required times by complicated limits. We could even see that the velocity comes out to be always touching the path, because the displacements taken to be smaller and smaller approach this direction. But all this is unnecessary, because, we could simply ask, how the body would move if from a point, all the future influences, that is forces would be removed. Clearly, it would keep on traveling in the straight direction and with the speed that it last acquired, that is with its last velocity.



Secondly, the fact that objects keep their velocities without any effort, justifies what we claimed earlier, that it's only the change of speed that counts, and it's immaterial how fast was originally the object. In fact, we can go even further and claim that if more objects are traveling with the same velocities, then they all "feel" compared to each other as if they were standing still. We definitely feel this when traveling on trains and can be completely unaware of our speeds to the outside world. On the other hand, we feel a bit uncomfortable about extending this logic to infinity. Indeed, it would mean that we could travel in a few seconds through the whole universe. As it turned out, from Einstein's Relativity, the velocities do have a limit, but this new logic required a whole rewriting of physics.

What's New About Newton

Some claim that Newton's big breakthrough was the introduction of forces. This is not correct! Obviously, people used some concept of forces already before. In fact, the balancing of forces had to be conscious in order to design buildings and simple machines, mills, and so on. It was also known that if the forces are unbalanced, then motions start, but usually this motion was not the main concern. When the Egyptians moved the big rocks to build the pyramids, they had to use tools to conquer, that is balance the huge weights, with smaller forces. Just balancing of course is not enough, we have to go over the weight a bit, so that the rock will move. But this over force or unbalancedness, was very small compared to the big forces that acted against each other. And indeed, all they needed was to move the stones. The speeds were not important, in fact the slower the blocks moved, safer it was. The concerns with speed only came later with trains and automobiles, much after Newton. So in a sense, Newton's mechanics was in its right time.
Newton's new concept of forces had two fundamental directions, namely how the unbalanced forces relate to motion and how the weight of bodies relate to forces in general. Both directions, had two fundamental realizations.
Firstly, if there are no forces, or rather there were no forces, then the bodies would simply keep their motions forever. This was mostly a theoretical statement to set the

right imagination for the real situations when the forces that act for a time, bring about changes of the speeds, depending on the masses of the bodies. This was the $f\,t = m\,(v - v_0)$ law which we still didn't put in its final vector form.

In the other direction, the first fact is that weight is just a force. Like any other, it takes part in the balancings, it causes the change of speed, like any other and most importantly it has to fight the resistance of the body like any other force. A second level of this direction was that if weight is just a normal force caused by planets, moons, and so on, then this force must apply to each other too. So just as an apple has weight that is a force towards the earth, the moon must have a weight too that is a force towards the earth. Similarly, the earth has a weight towards the sun, and so on. Then these planetary weights must be the causes of the planetary motions themselves, by the same laws that we'll find for the first direction, that is for the motions caused by unbalanced forces. So not only is weight merely a force, but planetary motions are merely motions by forces, ruled by the same laws as ordinary bodies.

Whether all these dropped into Newton's lap, gradually or at once, is not quite sure. The apple in the moonlight story makes it look like as if it all were just one big idea, namely that the moon must behave just like an apple. But I think, there had to be more breakthroughs in Newton's mind before the whole grand theory took shape.

Finally, there is a point that always goes unnoticed:

I mentioned that Kepler's mathematical oriented mind was essential in finding the planetary laws. Newton was very similar, but he thoroughly built from basics, so while for Kepler his laws were simply saying that God rules in the language of mathematics, for Newton, it was more that Nature speaks the language of mathematics. But this, to be meant in an even more specific way as follows.

For Kepler, there was a problem, namely how the planets move and there was a solution which turned out to be mathematical. For Newton, there wasn't one particular problem, of course when he was able to derive the Kepler laws everybody realized that he discovered something incredibly big. But we need too see much more to realize just how big this discovery was. The Kepler laws are just minor special consequences! The whole point of Newton's mechanics is that it is true as such in itself everywhere and for everything. This universality is the most important feature in Newton's discoveries, and strangely in both of the above mentioned directions. The unbalanced forces always and everywhere cause the same motions and the weights of masses, that is gravitation always and everywhere appears with the same law.

But it is just as important to realize what is not included in this universality.

Firstly, the laws of motions and gravitation are not all the laws of nature. This is quite evident by today, because everybody heard of other forces, electricity, and so on. Secondly, even if we raise some problem only about motions and gravitation, the laws themselves don't tell how to find the answer to our problem. It's up to human ingenuity to apply the laws of nature in order to find the answers to human questions. In other words, Newton created a framework between Nature and Man. Most importantly, all newer laws of physics discovered, remained in the same vain as what Newton initiated. This is a much better picture than imagining laws that describe nature.

A Horse Wagon Before And After Newton

A wagon has wheels which is an ancient discovery to overcome forces. It was also obvious already before Newton, that the horse works hardest at the beginning to start the wagon. This melted two initial difficulties. To start rolling the wheels against the road and to start moving the wagon itself. The two are not independent, because if the load is bigger, then it makes not only the resistance of the wagon itself bigger, but harder to turn the wheels too. So it's not surprising that the sheer resistance of the mass was not recognized. When a wagon is rolling, the horse still has to pull, because the wheels are going through rocks, the axels are not perfectly turning, even the air is against the motion.

We have two new levels of understanding the wagon's motion. The first law merely says that if the wagon is going with fix speed, then actually the horse is wasting all its force on the circumstances. The wagon would be able to move with fix speed by itself forever, if the circumstantial obstacles wouldn't slow it down.

The second law, $f\,t = m\,(v - v_0)$ tells that when the wagon is increasing its speed, it definitely needs more than a certain force. This minimal speed changing force is needed even if everything could be perfectly smooth and were no waste at all.

Toward A Full Picture By A Simple Law

There is something fishy about concentrating just on the wagon, and merely regarding everything else as disturbance. What's more, the horse is also ignored, even though his actions against the road are almost the opposite as of the wagon. Indeed, the road's friction is a loss for the wagon, but without it the horse couldn't even pull. So it seems like if there were some deeper meaning behind the disturbances. To shed some light on what's going on, amazingly we only need one new law. This is Newton's Third Law, and what it claims is so simple that we might think it's obvious too. Quite on the contrary, it leads to the most amazing consequences. An other unfortunate name for this law is the Law of Reaction. The reason of this name is that Newton called the $f\,t$ product of force and time as the action. Now, the Third Law actually claims that for any force exerted by an A body on an other B, there is an equal but opposite directional force exerted by B on A. Then of course, applying the same t time for this opposite force could indeed, be called the reaction. Still, this hides the actually moment by moment existing opposite forces. This opposite force should be called the counter force, and so Newton's Third Law is the Law of Counter Force!

The concept of A , B , . . . objects acting upon each other is not defined precisely and it is up to us how we pin point them. In fact, this whole law is most important for pushing us to look for the participating bodies and assign them with their masses and forces. For example, the road was a major obstacle in the wagon's motion and a helper in the horse's force exertion. So what should be the object, the whole road with the earth underneath or just the rocks and pieces separately? We have to make some idealizations to decide this, but these are quite logical.

The wagon usually conquers the little obstacles, so the road is not one object for it. Every little rock that the wheels must push aside are new interacting objects. On the other hand, the horse is better to be assumed as non slipping and indeed, its hooves are designed exactly for this. Thus, for the horse, the road indeed is actually the whole earth as one. The horse is the one that exerts an f force with its muscles, yet actually this force is between the pulling of the wagon and the pushing of the road. So the muscles are merely acting as springs between two objects. We don't care about the mass of the horse. In fact, the horse is not an object in our picture. Of course, in reality, the horse has to carry itself too, so its mass is important, but we ignore this because usually his force is mostly required for the pulling. Clearly a racing horse is different because there the big speed is important, at which even its own weight is a big task. So we should imagine the following:

The wagon is pulled with an f force, while the whole earth is pushed back with the same f, but in opposite direction. Of course, the earth's M mass is billions of billions times bigger than the wagon's m. Furthermore, under any t time, the earth is undergoing millions of other forces too. People blow up cars, do mining, or just jump around. But we can ignore these because M is so big that the $f\,t = M\,(v - v_0)$

makes $v - v_0$ practically 0 anyway. Quite on the contrary, the other forces to the m wagon, are not negligible! Little but many $m_1 , m_2 , \ldots , m_n$ masses as stones are in the way. Even the air could be regarded as tiny $m_{n+1} , \ldots$ molecules hitting the wagon. Every time one of these m_k masses suffer an f_k force from the wagon, an equal but opposite force will act upon the wagon itself. Then of course, with every little $f_k\ t = m_k\ (u_k - u_{k0})$ change of an obstacle, an opposite $f_k\ t = m\ (v_k - v_{k0})$ change of the wagon happens too. The natural picture is that the horse's f force is used for pulling the wagon and conquering the obstacles. Now this f and all other forces can be replaced by speed changes depending on the masses, but including the earth as the source too:

$$M (v - v_0) = m (v - v_0) + m_1 (u_1 - u_{10}) + m_2 (u_2 - u_{20}) + \ldots$$

When the wagon is sped up, then $m (v - v_0)$ is the major member on the right side. When it's going with fix speed, the rest of the members are the one that count only.

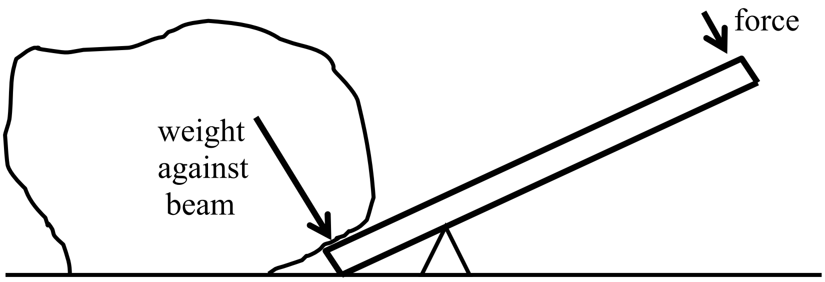
We can ask what happens to the $m_1 , m_2 , \ldots$ obstacles, that were moved by the wagon. By Newton's First Law, they should just keep their achieved $u_1 , u_2 , \ldots$ speeds, but this obviously won't happen in the real world. Indeed, each will encounter other masses, that are not listed above. So like an avalanche, the m v changes will spread further far beyond the wagon. Even the air molecules that were sped up by the wagon, collide with other molecules and give away their extra speeds and those again do the same. The full picture is really only the whole universe in which merely m v exchanges take place.

Something Is Still Missing

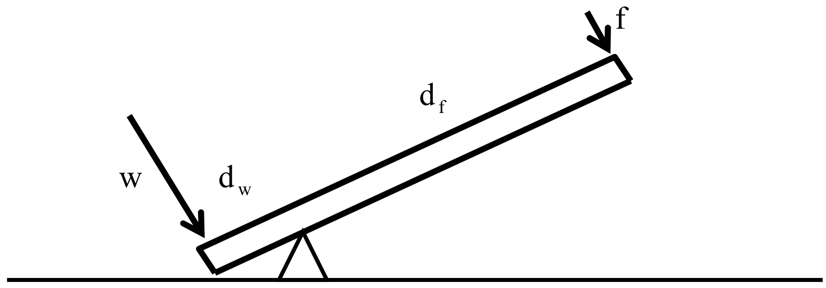
The replacement of forces with m v changes gives us the feeling that we were cheated somehow. Eliminating the horse from the picture of a horse wagon, just can't be the final solution. The horse was that motorized the whole avalanche of m v changes. So why don't we analyze the horse, go into how his muscles initiate the force between the earth and the wagon. We can even realize that his muscles can only do this if he was fed, if he is strong. So we are dealing with masses again. The food that he eats in the morning is essential for the force that his muscle will bring about. On the other hand, this at once shows that here we are dealing with something completely new because the $f\ t = m (v - v_0)$ law and the m v exchanges that followed from it only describe moment by moment simultaneous changes. This fact that the food is stored and changed into force later is something completely different. Maybe it's not the mass at all that is the crucial factor in such non simultaneous relations. After all, the horse needs special mass as food to charge up his muscles. In spite of this, and amazingly, there will be a bigger picture that again only involves the masses and rules all, even non simultaneous interactions. Such law, clearly will not be telling what food a horse eats, but it will still say that a certain mass of food, that is really consumed has a limit on the forces that the muscles can bring about. In other words, we'll have a new framework, wider than Newton's! Not surprisingly, it will be given by Einstein's Relativity, that went against the unlimited speed image of Newton's First Law.

Counter Force At Balancings

I mentioned that balancing of forces were known before Newton and were important for buildings and simple machines. Balanced forces of course, appear even at the most mundane circumstances. In fact, all the objects around us are usually in balanced rest. A few books lying on a table are a perfect example. The existence of counter forces, that is the Third Law, perfectly complements the Second Law. Instead of saying that the table doesn't allow the books to fall, we can say that the table is actually counter pushing the books, with a force that cancels the weight and thus, the total force is 0. Thus, the rest is not only following from the geometrical circumstances, but from merely the forces too. Unfortunately, sometimes these counter forces are applied incorrectly. For example, in our case, we can't just say that the counter force of the book's weight is the force with which the table is pushing the books. Indeed, the weight of the books is a force acted by the earth upon the books, while the Law of Counter Force talks about reversal between the same two bodies. So how does the table get involved? The table doesn't allow the books to fall, so they transfer their weights and act upon the table with this force. The table's push upwards is the counter force of this force. So now it's perfect, because the books push the table and the table pushes back. The reversal is between two objects. It might seem stupid to repeat the forces with these transfers, but that's the only way they all work out. Indeed, lets continue with the table. The table's weight is adding to the book's push because they are both downwards, so the table will transfer the sum of these and push the ground with this force. In other words, quite logically, the ground will be pushed with the combined weight of the books and the table. Though, this way of saying it is again incorrect, because actually the transferred sum of those forces push the ground. The ground counter pushes the table up with the total force. From this force the table only transfers the weight of the books as counter force up against the books. Of course, if we want to regard each book as a separate object, we can do that but then, that many force transfers and additions must be done in the weights and backwards as counter forces. In fact, in a continuous object like the legs of the table, we could continually place the increasing weight and the counter force. This may sound weird, but actually quite logical because the tension in an object is all along and is caused by the everywhere balanced forces.
The simplest machine is the lever. We want to lift a heavy stone, so we slide a rigid beam under it and then rest the beam on a strong pivot, close to the stone. The other side of the beam is much longer and at its end we can push it down and thus lift the stone with an "amazingly" small force:

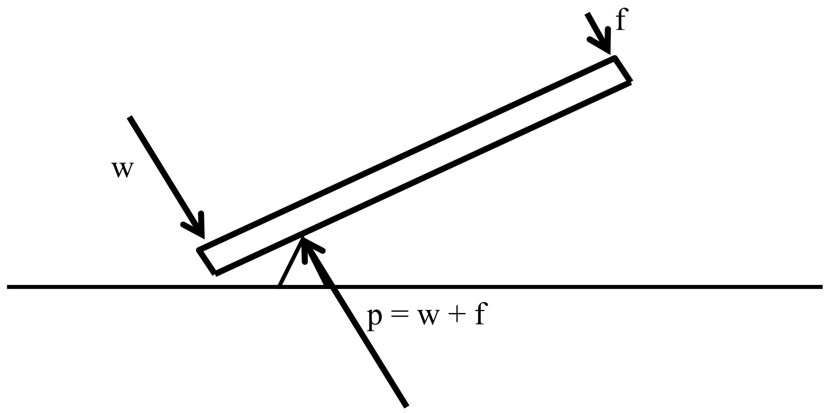


Again, lets realize that it's only the balancing that is important here, even though actually we want to roll the stone away. Indeed, we have to over do a bit the weight, but it's immaterial how much. Once the stone is balanced, any small extra force is enough to lift it a bit and put logs under it or use other tricks. We are not in a hurry, we don't want to speed up the stone, just move it a bit with practically zero speed. People even realized that the w weight against the beam can be balanced with an f of the same proportion to w as of the two arms of the lever.



$$\frac{f}{w} = \frac{d_w}{d_f} \quad \text{or in a better form} \quad w\, d_w = f\, d_f .$$

The f force could be called the "counter force" of the w weight by everyday logic, but lets remember that we used this expression for a very specific purpose, namely for the always appearing opposite forces between objects. These two, w and f couldn't be counter forces for three obvious reasons: Firstly, they both act on the same object, namely the lever. Secondly, they are not equal in size. Thirdly, they are not opposite in direction. So, where could the counter forces be? Well, we forgot about the pivot point. That's where the counter force of both w and f appear. But how? The combined force of w and f, indeed pushes the pivot, but down again in the same direction, so with this we are even worse, having double of the forces without counter force. We jumped too much ahead! Lets remember that we have to tell the objects on which and by which the forces act. The w and f forces act upon the lever. Their sum is indeed, transferred by the rigidity of the beam, but that acts upon the pivot or actually the whole earth. Of course, this force has a counter force and this is the p force of the pivot which acts upon the lever as the third force, the combined counter force of w and f.



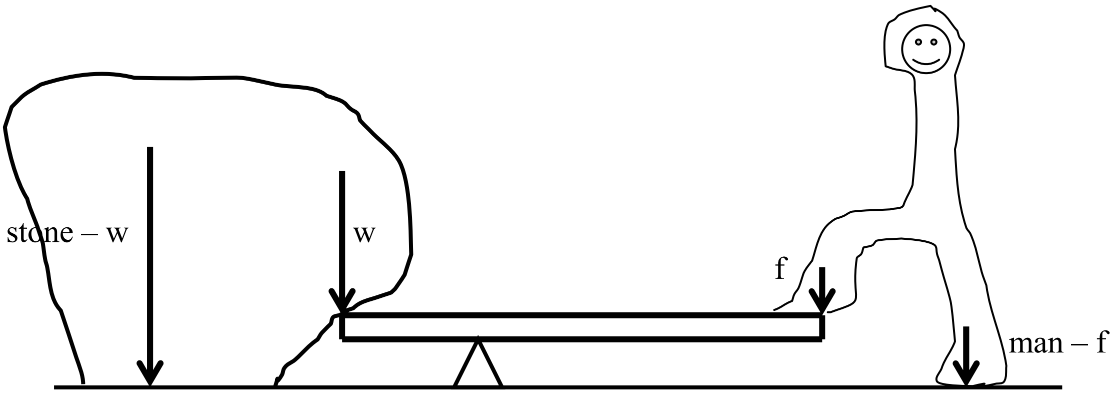
So the total force on the lever became zero.

Counter Weight

In both of our examples, the books lying on a table or the lever, the counter forces of weights were merely introduced to satisfy Newton's Third Law. If we push an object together with our two hands, then the force and counter force both have natural source as our muscles. Amazingly, the counter forces of weights are just as natural, namely they are weights themselves. In short, they are counter weights. Of course, just as the "counter force" had its improper everyday usage as we explained above, the "counter weight" is even more frequently used as weights applied to balance a container, a boat, or car.

What are weights? Attractions of masses towards planets, moons and so on. The Law of Counter Force must be applied to this too: So when a mass, like an apple, is

attracted by the earth, then the same but opposite force, must also act upon the earth. Two equal, but opposite forces appear, one by the earth on the apple, and one by the apple on the earth. The first we already knew and called the weight of the apple by the earth. So then, the other should be called the weight of the earth by the apple. This sounds very weird, but with a deeper logic, can become quite natural:
Why should the planets be special at all? Can't it be that actually all objects in the universe have an attraction to all other objects? But then, why don't we see objects falling towards each other in our room? Because they are too small! The attractive forces between them are so small that the circumstances like the air conquer them easily. The earth's huge mass on the other hand is enough to create enough force to bring about the weight of the apple. But this is still a wrong picture! It is much better to imagine the earth as made of little masses like billions of apples. Each of those imaginary apples and the real apple have equal but opposite minute attracting forces towards each other. Then of course, the billions of minute forces acting upon the real apple add up to a measurable force, which is the weight of the apple. But also the billions of minute forces of the imaginary apples add up because they are stuck together as the earth. Then the only question is where this common weight of the earth by the apple should be placed. Usually, we imagine it at the center of the earth, but through the rigidity of the earth itself, it can be placed anywhere in between even at the surface.
So the counter force of weights is the gravitational counter force. Amazingly, the proper distribution of these counter forces is automatic! The table and the books attract the earth exactly with their combined weights. We could place this in the center of the earth or anywhere in between. In fact, this combined force is the one that pushes the table up from the ground. But on the table, only the weight of the books will remain because, the continuously increasing weight of the table as we go higher and higher, is an other force reducing the push of the ground.
Earlier we said that the table's push towards the books can't be simply regarded as the counter force of the book's weight, because the book's weight are from the earth so the table is not even involved. To involve the table, we realized that it doesn't allow the books to fall and thus the books will transfer their weight as a "new" force upon the table. Then of course the push upwards is the perfect counterforce of this.
Now it turned out that actually we were not that wrong to start with. The weight of the books also has its counter force, of course upon the earth as a weight towards the books. But then this force is transferred through the earth and the table as the table's counter force. In short, the source of the table's pushing as counter force is actually the counter force of the weight that caused the push on the table.
An apple hanging from the tree is practically the same as the books lying on the table, except here, the counter force of the weight is coming from the top through the stem and the tree. Still the line of forces and counter forces can be continually placed as the tension in the rigidity, that keeps the forces in balance and the object in rest. If there are any points too weak to handle this tension then there the chain breaks. Usually this is the stem. At that moment when it breaks, the apple's w weight is not equalized anymore, it will have an unbalanced weight and will speed downwards according to the $w\,t = m\,(v - v_0)$ law. The earth will have the same unbalanced weight and will "speed" upwards by $w\,t = M\,(\nu - \nu_0)$. Of course, M is huge, so ν is minute. The t time is only until the apple reaches the earth and equalizes its weight again, but now through the ground. Under this time the earth would move practically nothing. The whole argument is theoretical anyway, because under this t time there are millions of other forces around the earth.
Lets return to our w and f forces of the lever! For simplicity, lets imagine them now completely vertically and the lever horizontally:



The stone's push towards the ground will be decreased by w while the man's push with f. These reduced weights will have their counter forces through the ground, while the w + f part of the total weights will be equalized through the pivot. So the earth's gravitational counter force will be distributed, but they all come from the weight of the earth by the stone and the man. Finally, a remark:
Our imaginary apples that were attracted by the real apple were combined into one force. We placed this in the center or along the line between the center and the real apple. If we had a more exact prediction about the gravitation between objects that depends on their distance too, then it would be a completely new problem whether the imaginary apples can indeed be combined into the center of the earth. In other words, would the earth's gravitation be the same on a real apple if we compressed the earth into its center, but keep the apple at the same distance. Then of course, the earth's radius is enough to calculate the gravitation at its surface. As we'll explain it later, this became a big problem for Newton and only after long struggle was he able to prove that indeed, the gravitational effect of a ball can be placed in its center.

Total Force

The Law of Counter Force, through its unrelenting application even for gravitation, lead to the beautiful result that when the forces are balanced their sum is zero. At our lever it meant that $p = w + f$, so the pivot balanced the weights. But lets remember that the whole point of the lever was to overcome w with a smaller f. Or the wider question was how w and f relate depending on the point of the pivot. The $p = w + f$ equation doesn't say anything about this. This ignorance of the real problem and rather to get a bigger picture, is similar to the earlier grand view of m v exchanges, by ignoring the horse, that is how forces arise at all. There we ignored something that is complicated, but here the lever ratios were simple and known very well, yet now we don't care about it. That's true, in fact we go further and combine the two bigger pictures into an even grander view:
Before we said that in $f\,t = m\,(v - v_0)$ the f force is the unbalanced one. Now, we know that in balancings the total is zero, so we don't even have to go into the distinction whether the forces were balanced or not. We can simply say that f in $f\,t = m\,(v - v_0)$, is the total sum of all forces.

By the way, even in our horse wagon picture, many balancings were involved too. In fact, the wheels are continually operating levers, but now we have the justification to ignore those details too.

Where Will The Frameworks Lead Us?

We said that the new concept of laws, that Newton brought to mankind is the change from details into bigger frameworks. We saw this in action very well! Horse is ignored, law of levers is ignored, all this to get more abstract and general statements.
Are we going in the right direction or maybe are we just fooling ourselves? It all boils down to whether we will return to the details or not. Will we know how the horse's muscles work? Will we be able to derive the already well known rules of the lever arms? The answer is yes!
We already mentioned the wider framework of Relativity. That will be a drastically new system, surpassing even the one created by Newton. But smaller refinements of Newton's framework already lead back to the seemingly ignored details. Most important is to see the difference between our ignorance of abstraction and the ones that occur in metaphysical arguments. When in physics we ignore a detail, we are completely aware of it! We don't pretend to solve it by going above it. In fact, we hope that we'll obtain some new information that will lead back to it. Metaphysics operates quite differently, the problems are not objectified, so the wider thoughts as solutions are not objective either. It doesn't mean that metaphysics is always faulty. In fact, it precedes all physics because we have to have a gut feeling of what a problem is before we objectify it and then we also go by our gut feelings how to find the new frameworks.
When Newton said that "physics beware of metaphysics" he merely warned about this danger. Later, Hegel replied that Newton practically said "physics beware of thinking". That's true, because indeed, all thinking is metaphysics. But it was a witty or sarcastic truth and thereby making the original statement of Newton ridiculous and so appear false. Yet in fact, the two truths are not in contradiction. Yes indeed, "physics beware of thinking" is just as correct. Or to put it in an even sharper way "physics beware of stupidity" is obviously true, though all stupidity is thought too.
So then physics must have a guideline of being true, besides the thoughts that lead to it. Most would at once say that this guideline is experiment. And indeed, to be a bit wider, we have to admit that observable and repeatable facts always conquer all opinions. But understanding is an other changer of opinions. Unfortunately, understanding is usually not a goal of individuals and as a rule is not the goal of society.

Falling Up Or Down

Still without the vector meaning of the force, that is without the change from f to F, we can regard the very special case, when the weight force is used in only up and down directions. Galileo dropped objects, that is let them fall from 0 initial speed. We can be a little bit more general and allow that an object is thrown down or even more interestingly, thrown up with an initial v_0 speed. The question of course again is, what will its speed be after t time and even more importantly, what distance will it travel. In the fundamental $f\,t = m\,(v - v_0)$ equation, now we have to use the force of weight, which we claimed to be proportional to the mass itself. So, $f = m\,a$ with a constant a, that is particular to the planet, sun or moon, where we are. The earth's weight constant is usually denoted as g and its value is 9.8. But to stay general:
$m\,a\,t = m\,(v - v_0)$ implies $v = v_0 + a\,t$. In other words, the speed is increasing with the a constant multiplied by the time. This usually is called the acceleration and thus, a planet's weight constant is also the constant with which the falling objects accelerate when dropped or thrown down. In the throwing up case, that is when the speed is decreasing from the initial v_0, we must use $m\,(v_0 - v)$ and so we get the

alternative $v = v_0 - a\,t$. So the weight constant is also the slowing down factor or so called deceleration, for objects thrown up.
The two cases can be combined as $v = v_0 \pm a\,t$ or constantly changing speed.

Calculating The Distance

Now we'll show how to find the traveled distance of motion with constantly changing speed. We cut the t time interval into n equal parts and under each little $\frac{t}{n}$ interval, pretend that the motion is with a fix v_k speed. So, from the first $\frac{t}{n}$ time interval, we choose a v_1 speed and regard the motion with this fix speed and then the traveled little distance would be $d_1 = v_1 \frac{t}{n}$. Then from the second little $\frac{t}{n}$ time interval, we choose a v_2 and with this, kept fix, the distance were $d_2 = v_2 \frac{t}{n}$.
And so on, we can approximate the real d as the sum of these little fix speed trips, so:
$d \approx d_1 + d_2 + \dots + d_n = v_1 \frac{t}{n} + v_2 \frac{t}{n} + \dots + v_n \frac{t}{n} = \frac{t}{n}[v_1 + v_2 + \dots + v_n]$
If we choose the initial speeds in every interval, then:
$v_1 = v_0$, $v_2 = v_1 \pm a \frac{t}{n}$, $v_3 = v_2 \pm a \frac{t}{n}$, ... , $v_n = v_{n-1} \pm a \frac{t}{n}$
Lets write these speeds again, but in reverse order:
v_1 , v_2 , ... , v_n
v_n , v_{n-1} , ... , v_1
Since each v_k is a $\frac{t}{n}$ bigger or smaller than the next or the previous, thus all two speeds under each other add up to the same value:
$v_1 + v_n = v_2 + v_{n-1} = \dots = v_n + v_1$. Thus, the two sets of speeds, together give $(v_1 + v_n)\,n$ and so just one's sum is $\frac{(v_1 + v_n)\,n}{2}$.
Here, $v_1 = v_0$ and $v_n = v_0 \pm (n-1)\,a \frac{t}{n} = v_0 \pm (1 - \frac{1}{n})\,a\,t \approx v_0 \pm a\,t = v$, for large n. Thus, $d = \frac{t}{n} \frac{(v_0 + v)\,n}{2} \approx \frac{v_0 + v}{2}\,t$. This is very plausible if we realize that $\frac{v_0 + v}{2}$ is the average speed and so the actual distance can be calculated as if this average speed had been the fix speed for the full t time.
If we choose not the initial, but the last speeds in each interval, then:
$v_1 = v_0 \pm a \frac{t}{n}$, $v_2 = v_1 \pm a \frac{t}{n}$, $v_3 = v_2 \pm a \frac{t}{n}$, ... , $v_n = v_{n-1} \pm a \frac{t}{n}$
Thus, $v_n = v_0 \pm n\,a \frac{t}{n} = v_0 \pm a\,t = v$ and if n is large, $v_1 = v_0 \pm a \frac{t}{n} \approx v_0$.
So again, $d \approx \frac{v_0 + v}{2}\,t$. This also means that if we had chosen other v_k values in between the initial or last speeds, then we would have gotten intermediate approximations which thus must also give the same value. In short, the choice of speeds from the little intervals are immaterial.

The $v = v_0 \pm at$ and $d = \frac{v_0 + v}{2} t$ equations perfectly describe the motions with constantly changing speeds. In other words, if from the five data: t time, v_0 initial speed, v final speed, a acceleration and d traveled distance, we give any three, then the missing two can be calculated from the equations.
In the simplest case of dropped objects, $v_0 = 0$ and thus,

$v = a t$ and $d = \frac{v}{2} t = \frac{a t}{2} t = \frac{a}{2} t^2$.

On the earth, the $a = g = 9.8$ can be approximated as 10 and so $d = 5 t^2$.

Partitioned Product

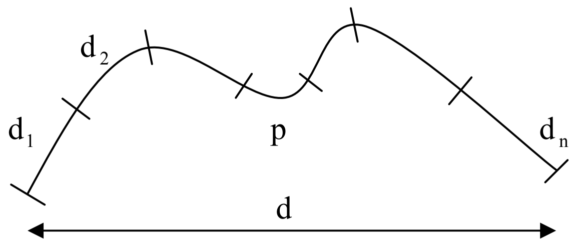
Lets repeat the main idea that we used to calculate the distance. First of all, we know that if a v speed is fix, then the distance in t time is $v t$. Of course, usually v is changing under the t time, so we cut t into little $\frac{t}{n}$ intervals, choose v_k speeds in them and calculated the little distances with these fix speeds as $d_k = v_k \frac{t}{n}$.

Then: $d \approx d_1 + d_2 + \ldots + d_n = v_1 \frac{t}{n} + v_2 \frac{t}{n} + \ldots + v_n \frac{t}{n}$. This sum of products could also be called as the partitioned product of the speed with the t time. Indeed, we couldn't calculate d as $v t$ because v was not fix. So we simply partitioned t into segments, used fix v-s in each segment and added them up.
We will even use the $v \circ t$ symbol for this sum to emphasize that it is basically a product, merely replaced by sum of products to get the best approximation for it.
In other words, we should regard $v \circ t$ as a best compromise to get $v t$!
If v is a constant value, then $v \circ t$ is the same as $v t$. If v has two values, v_1 under the first t_1 time interval and v_2 under the next t_2, then $v_1 t_1 + v_2 t_2$ is clearly the new refined replacement for $v t$. Of course, if v is continually changing, then we have to make the partitionings, but the same logic applies.
It's important to see though, that in the $v \circ t$ symbol, v and t are not independent. They don't mean anything separately and we are not dealing with a product of them. That was the whole point, that we couldn't calculate the product from two members. On the other hand, if we get a value for $v \circ t$ as a whole, then we can divide this with the t time and this $\frac{v \circ t}{t}$ value could be denoted separately as $\bar{v}$, which is the average of the changing v. In the case of constantly changing speed, $d = v \circ t$ became $\frac{v_0 + v}{2} t$ and thus, $\bar{v} = \frac{v_0 + v}{2}$. In other words, the average v became an other meaning of the average v, namely the simple middle value. If v is changing, but not constantly, then the only meaningful average value can be obtained as $\frac{v \circ t}{t}$.

Partitioned Product Of Velocity, Ignoring The Directions

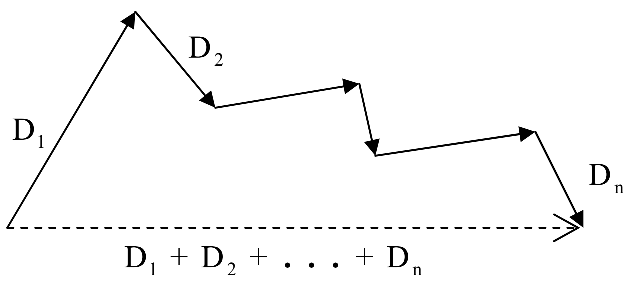
The $v \circ t$ partitioned product can just as well be calculated when instead of a one directional motion, we have V velocities with different directions at every moment. Picking the V_k velocity in each small $\frac{t}{n}$ time interval, of course now would mean a picked direction too, but if we ignore this direction and only use the $|V_k|$ length,

then the $|V_1|\frac{t}{n} + |V_2|\frac{t}{n} + \ldots + |V_n|\frac{t}{n}$ sum will only give a number. Most amazingly, this number won't be the traveled d distance anymore, but an even more useful value, namely the actual p length of the path.
Indeed, $d_1 + d_2 + \ldots + d_n$ approaches p, not d. In short, $|V| \circ t = p$.



Partitioned Product Of Velocity With Direction

The obvious question is whether we could calculate a $V \circ t$ value, that is keep the little V_k as vectors. For this, we should give a D_k vector meaning of $V_k \frac{t}{n}$ and then be able to sum the $D_1 + D_2 + \ldots + D_n$ vectors into one.
Both can be achieved very naturally!
A V vector multiplied by an x number should simply be the same directional vector as V and only its length changed from $|V|$ to $|V|x$. In other words, the multiplication with a number simply extends or shrinks the vector. Of course, in our case the multiplying numbers are $\frac{t}{n}$, so as n increases it will mean smaller and smaller $D_k = V_k \frac{t}{n}$ vectors. Now for the summing of vectors, lets imagine what a $d_1 + d_2 + \ldots + d_n$ number sum means. We simply place the $d_1, d_2, \ldots, d_n$ distances after each other. So then obviously, for $D_1 + D_2 + \ldots + D_n$, we do the same but now with changing directions, that is chain the $D_1, D_2, \ldots, D_n$ vectors together:



Amazingly, now $V \circ t = D$ is obtained, that is the partitioned product of the real velocity with time doesn't give the length anymore, rather the displacement vector under the t time. The $|D|$ absolute value of this is the d traveled distance.

The Three Levels Of Vectors

We should observe that the approximation with changing directions is much more questionable than before with simple numbers. Indeed, every little $D_k = V_k \frac{t}{n}$ vector must be moved a little bit to be able to chain them together. Then it's not obvious in what order, that is with what priority we should move them. Of course, we can see that as all D_k become smaller and smaller, then these ambiguities become less and less important. A drastic help to achieve the summing of chaining of vectors is if we simply regard parallel shiftings as merely identical variants of a vector. The real question is whether this is physically justified or not.

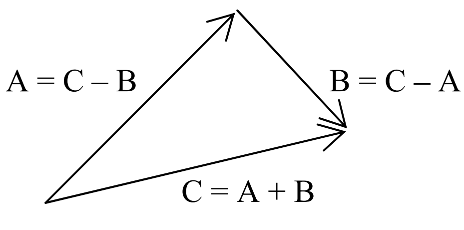
In some physical applications, a vector is just identical to itself in its actual sense as the connecting arrow of the two end points. In some applications it doesn't matter if we move it along in its own line. This is clearly what Newton's First Law used for the velocity, because the body moves in one line. Finally, sometimes even sliding of the vector sideway doesn't matter and usually this most free meaning is what we use for the mathematical vectors.

As far as forces are concerned, we didn't use them as vectors yet, but we can already see that all three levels can be meaningful. If we apply a force to a real body with flexibilities, then we can't allow even the sliding of the force in its own line, because it might simply bend at one point, while push at an other. If the body is perfectly rigid everywhere, then it turns out that the shifting of the force in its own line wouldn't make any difference. Of course, shifting sideway still can't be allowed because pushing an object near its perimeter is causing much more turning than applying the "same" force closer to its center. And finally, if we disregard the turnings and only care about movements of bodies as if they were points in space, then we might argue that a point moving in space is the same as the whole space moving backwards, so then it's more convenient to regard even parallel side shifted forces as identical.

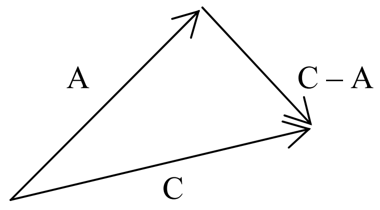
The total shiftability of mathematical vectors makes many convenient consequences. For example, it implies at once that in the chain sum, the order of addition, that is chaining is unimportant, just like at numbers.

The Triangle Rule Of Vectors, Coordinate Location

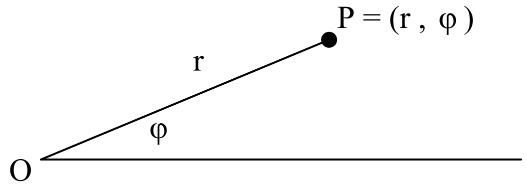
The letter D for the displacement vector as the result of the $V \circ t$ partitioned velocity product with time could be regarded as "D for disappointment" too. After all, the simpler $|V| \circ t$, obtained from merely the speed, already gave the more important actual p length of the path. So going through vectors just to get the D connecting vector from the initial point to the last, is really much ado about nothing. Yet, we are wrong! D is more important than p. To appreciate though this displacement, we need two new concepts. The first, is merely the reversal of vector addition, that is vector subtraction. The word "reversal" already helps, because if we know that $A + B = C$, then we should expect that $C - A = B$ and $C - B = A$. This of course is purely algebraic reasoning and we should check it out with our chain rule addition:



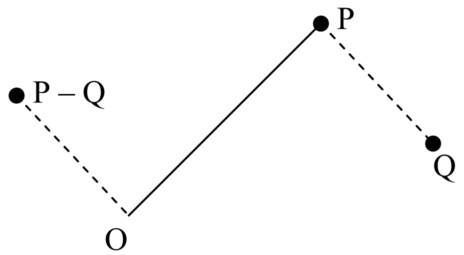
As we see, the $C - A$ and $C - B$ subtraction vectors are also "short cuts" of the two other members, like C was from A and B. But while A and B were in a chain so that the short cut was meaningful as the traveled distance, here quite oppositely the other two either start together like A and C, or end together like B and C. Furthermore, while at $A + B$ the direction was the obvious towards the direction of the motion, that is from the tail of A to the arrow of B, here the directions must be learnt! Namely, at the common starting A and C, the $C - A$ goes towards C, but at the common ending B and C, C – B goes towards B. The name Triangle Rule is quite obvious from the picture, but usually only the common starting case is regarded:



The second concept we promised explains why the common starting vectors was the important case above. The Descartes Coordinate System is usually introduced as two perpendicular number lines by which we can locate the points of the plane. It's rarely mentioned that other ways of locating points could be achieved too! For example, we can measure the distance of P from a fix O point and the angle of OP to a fix line through O:



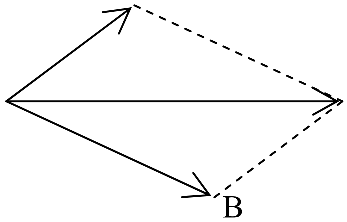
Here the two coordinates are r and φ, unlike x and y in the Descartes system. With the use of vectors, all coordinate systems could be treated together. Indeed, we merely choose a fix O point, and then any P point of the plane or even in space, can be regarded as the vector going from O to P. In some modern geometry books, they start right away with vectors, jumping through points at all. Or we could start with points, but regard them as vectors, that is we don't use arrows for them, but we define the addition and subtraction of points themselves. For example:



As we see, the parallel shifting is included in this concept. No matter what approach we choose, to emphasize or hide vectors, at the end it all boils down to the following simple fact: When motions are described in a coordinate system, that is as P (t) locations relative to a fix origin, then $V \circ t = D = P(t) - P(0) = P - P_0$.
So we get an equation for the locations.

Parallelogram Rule Of Vectors, Combining Of Forces

If all vectors are measured from a fix O origin, then the chain rule itself should be replaced by a direct way of seeing the sums of vectors that start together.
This direct way is called the parallelogram rule.
Indeed, if A and B start from one point and we draw parallels to both vectors through their arrows, then these parallels cross exactly where A + B should point:



This is obvious if we regard the parallel shifted versions of B or A, that make a chain sum exactly to A + B.
This parallelogram rule unfortunately only works for two members. If we have more than two members, we have to use the rule repeatedly.
The parallelogram rule is especially useful for adding up forces that act upon one body. Two ropes pulling a body with different forces is a perfect example.

Force As Vector

The $V \circ t$ partitioned product of V with time, can be used for any vector in place of the velocity, but then of course, the result will not be the D displacement.
The $f\,t = m\,(v - v_0)$ equation suggests how to generalize it to vectors. On the left, we use F vector instead of f, and calculate $F \circ t$, while on the right merely replace the v speed, with V velocity! The $V - V_0$ subtraction of vectors must be meant by the triangle rule and the $m\,(V - V_0)$ as simple stretching or shrinking by the m numbers. Now, the triangle rule clearly shows that if V and V_0 are stretched, it leads to the stretch of the whole triangle, so $m\,(V - V_0) = m\,V - m\,V_0$, just as we would expect it. So, we have our equation as: $F \circ t = m\,V - m\,V_0$. The separate application of m is logical because then the $V \circ t = D = P - P_0$ is parallel completely, having again a change of vector on the right.

Effect, Momentum, Impulse

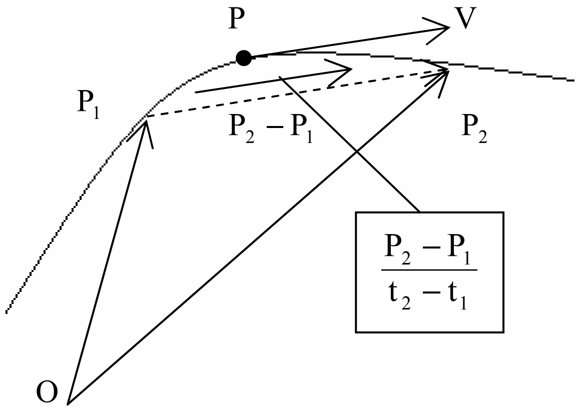
Lets call the partitioned product of a vector with time as its <u>effect</u>!
Then, the $V \circ t = D = P - P_0$ equation means that the effect of velocity is the change of location.
Lets call the $m\,V$ vector, that is the velocity proportionally increased by the mass as the <u>momentum</u> of a body!
Then the $F \circ t = m\,V - m\,V_0$ equation means that the effect of the force is the change of momentum!
The word “momentum” is very logical as the ability of a body to hit something. This is clearly proportional to its speed, and also to its mass.
The $m\,V - m\,V_0$ change of momentum is usually called the impulse. An other word for it could be impact. But both words are misleading, because they don’t reflect that $m\,V - m\,V_0$ is also a vector. If for example, $|V| = |V_0|$, that is only the direction of V_0 changed, then still a definite $m\,V - m\,V_0$ impulse or impact happened. But to ask if this impulse was given or taken by the body is meaningless.

Changing Mass

Beside this new wording of the Second Law, a much more important real generalization is hidden behind the separate $m\,V - m\,V_0$ application of the mass. Indeed, we can imagine that just as the V velocity of the body, its mass is changing too. Then m is not a fix number anymore, rather a function of time and if m_0 is at the beginning and m after t, then: $F \circ t = m\,V - m_0\,V_0$
This is now finally the Second Law of Newton.
The changing of the mass could be achieved for example, by injecting some liquid into a cavity through a tube. Then, its possible that we apply a force and it will not increase the speed at all, rather "be used" for moving the new mass. Still, the effect of F, that is $F \circ t$ will be $m\,V - m_0\,V_0$. Even stranger is if we don't apply any force, just increase the mass. Then $F \circ t = 0 = m\,V - m_0\,V_0$, so as m increases from m_0, V will shrink from V_0. This could be regarded as a new altered form of Newton's First Law for changing mass.

Change Rate And Its Effect

The $V \circ t = P - P_0$ equation means that the effect of velocity is the change of location. Is this a physical law or a mathematical? The use of coordinate system for the right side, may suggest that it is mathematical, but the velocity given moment by moment as the measurable direction and speed with what the body would move if let go, suggests that it is physical. Could we eliminate this physical meaning and get V from the locations too? It's quite easy! If the locations at t_1, t_2 moments are P_1 and P_2, then $\frac{P_2 - P_1}{t_2 - t_1}$ is an average velocity between the two positions. If both t_1 and t_2 approach a t moment of time, then this average will approach the actual V at t. This can be seen on a path too:



The crucial fact is that though both $P_2 - P_1$ and $t_2 - t_1$ become smaller and smaller, their division will not! In other words, the $P_2 - P_1$ vector would shrink to nothing, but being divided by the smaller and smaller $t_2 - t_1$, it is actually increased and thus, it keeps its direction and even approaches a limit length.
Lets call the $\frac{P_2 - P_1}{t_2 - t_1}$ limits in general as change rates! Then the velocity can be called the change rate of the location and the $V \circ t = P - P_0$ equation, becomes purely mathematical, saying that the effect of the change rate in location is the change of location. Of course, the location was just a vector here, so we have it in quite general: The effect of change rate is a change.

Newton-Leibniz Rule, Subconscious Integration

This in fact, is the fundamental law of calculus, the so called Newton-Leibniz rule. It expresses that the two limit calculations, the change rate or differential ratio and the effect or integration are in harmony, so applying them after each other, gives back the original quantity or at least its change.
Amazingly, all people use this law without being aware of it. In fact, when we ask someone “How was your day?”, we expect the person to use it. To be more specific, lets imagine that the person runs a shop. Now a perfect way to tell the total sale, is by counting the money in the cashier. But as the customers appear, the boss has a direct moment by moment feel of the sale too. At the end of the day, he will remember this moment by moment change rate of cash flow, the peaks and lows of customer traffic and even how long they lasted. So he can estimate the total, with the same method that we used, that is adding up the $v_k\, t_k$ changes, where v_k are the change rates and t_k are their lengths.

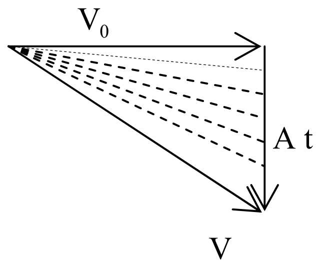
Acceleration Vector

Just as the velocity’s effect gives back the change of location, that is $V \circ t = P - P_0$, the velocity itself or its change could be an effect of something, namely of the velocity’s change rate. But what is the change rate of velocity? It is what we normally call acceleration, but now $\dfrac{V_2 - V_1}{t_2 - t_1}$ is a vector, so the directions can be very surprising. Still, we can call this as A and then $A\,t = V - V_0$.
This is a purely mathematical equation!
On the other hand, by the Second Law, $F \circ t = m\,V - m_0\, V_0$.
If we only regard one body’s motion with a fix m mass, then from this,
$\dfrac{F}{m} \circ t = V - V_0$. Thus, $A = \dfrac{F}{m}$, that is the F force on an m mass, brings about exactly this acceleration “F on m”. So if we are confused about what the exact mathematical definition of A would give, we can rely on F.
Nowadays, Newton’s Second Law is usually given as $F = m\,A$.
This is not only incorrect historically, because as we said, Newton used the $F \circ t$ effect of the force, which he called the action, but also hides the real meanings and makes it look like as if the A acceleration vector would be something quite obvious. Even more importantly, the $F = m\,A$ form is not as general as $F \circ t = m\,V - m_0\, V_0$.

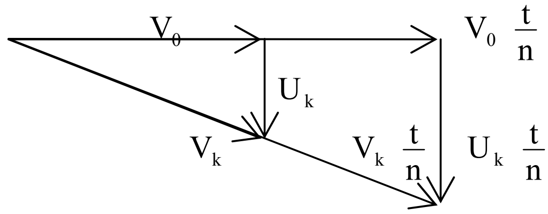
Falling Bullet

After half second, a dropped object falls, $d = 5 \cdot \left(\dfrac{1}{2}\right)^2 = \dfrac{5}{4} = 1.25$ metres.
That’s about the height that we usually fire a gun.
So if we drop the bullet from this height, it falls down in half second. We would tend to believe that if instead of dropping the bullet, we shoot it straight ahead, then it would land much later than half second. But this is wrong! The decrease in height is the same in time as would be at simple dropping. On the other hand, the forward motion goes on with the fix speed of shooting just as it would be in empty space. It only stops because the falling makes it reach the ground.
This fact is usually referred to as the independence of coordinates, meaning that the horizontal motion is not affecting the vertical falling and vice versa, except at the

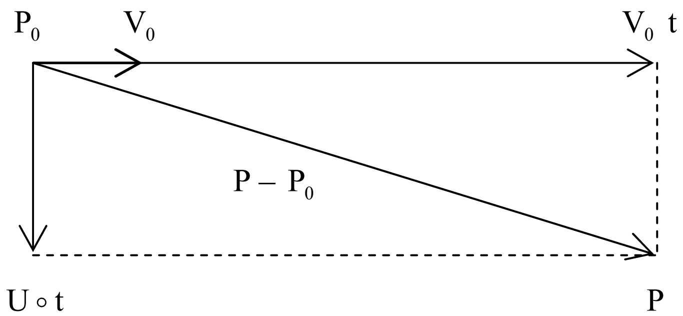
landing, as an obviously common stop. Unfortunately, the stating of independence stops here and thus making it look like as if itself were an "independent" law. This is false, and it is actually the simplest special consequence of the new vector meaning of Newton's Second Law. The weight is an F force downwards that is proportional to the falling mass, so $\frac{F}{m}$ is a fix A vector. Just as before the fix a value, now this A vector is the common acceleration of all falling bodies. But now, if the body is launched in any V_0 velocity, not just down or up, we can obtain V after a t time from $A \circ t = A\,t = V - V_0$ as $V = V_0 + A\,t$. In particular, if V_0 is horizontal, then V is merely the biggest side of the V_0 fix and $A\,t$ increasing sided right angle triangle:



But that's not all! The locations themselves can be obtained from the changing velocities as their effect: $V \circ t = P - P_0$. And here is the crucial point!
All the in between $V_1, V_2, \ldots, V_n$ velocities must be multiplied with their little $\frac{t}{n}$ time interval. But looking at $V_k \frac{t}{n}$, we at once see that it is the same as $(V_0 + U_k)\frac{t}{n} = V_0 \frac{t}{n} + U_k \frac{t}{n}$ where U_k is the vertical side of the triangle:



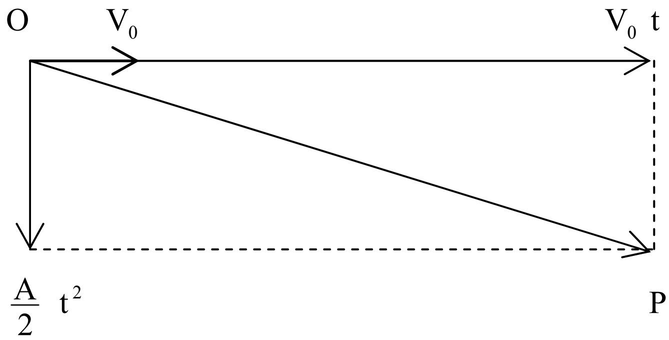
So summing these members the same will hold and thus:
$V \circ t = V_0 \circ t + U \circ t = V_0\,t + U \circ t = P - P_0$. Here $U \circ t$ is the simple vertical fall.
The independence can be seen even better if the $U \circ t$ member is shifted to the initial P_0 point and we use the parallelogram rule instead of the chain:



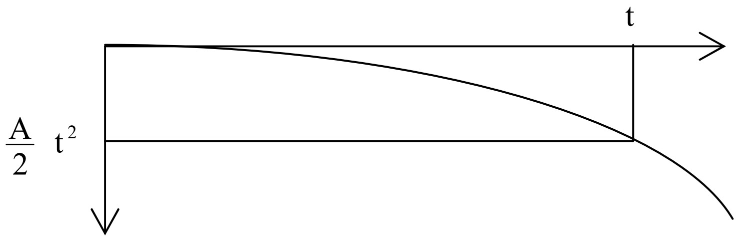
The $U \circ t$ vertical fall from 0 initial speed, can be calculated as before, that is:

$$U \circ t = \frac{0 + U}{2}\,t = \frac{0 + A\,t}{2}\,t = \frac{A}{2}\,t^2 .$$

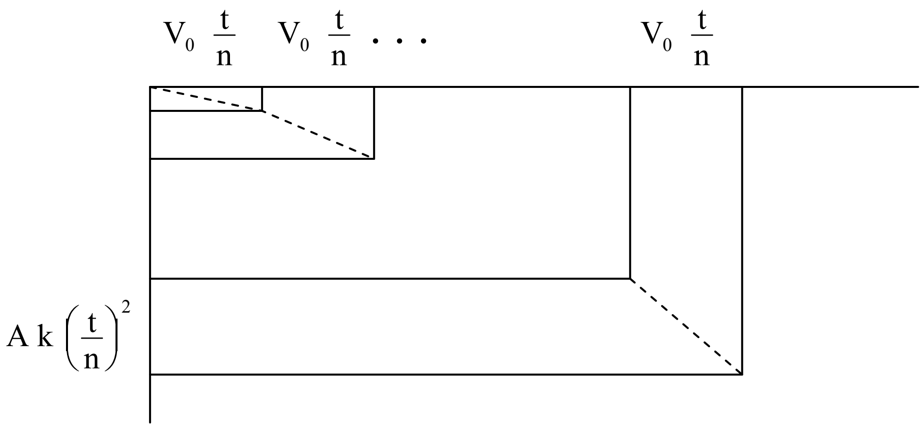
If we choose P_0 to be the origin of the coordinate system, then:



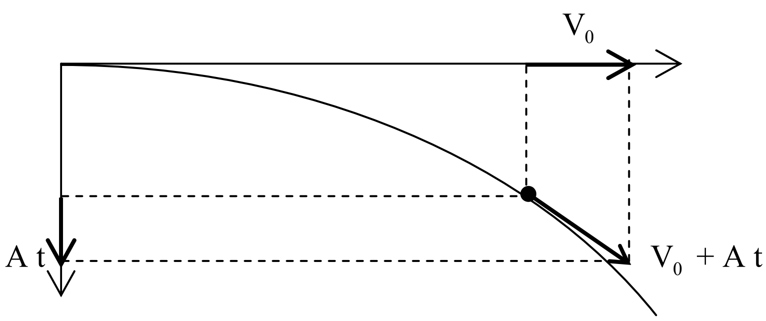
If t is continually followed then, the P positions go through an upside down parabola:



This was merely used here as a mathematical fact. But we can go physically too!
Then the t time should be cut into little $\frac{t}{n}$ intervals and in each, the motion approximated as a linear one with $V_k = V_0 + A\,k\,\frac{t}{n}$ velocities:

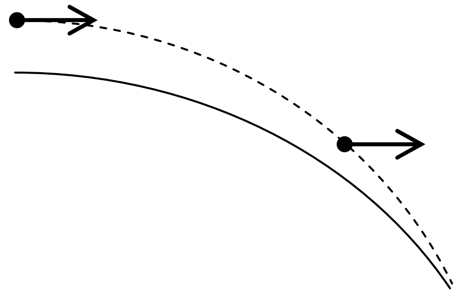


This when refined will lead to the parabola.
Or we can even return to the physical meaning from the final path! Indeed, the moment by moment velocities must be on the path as the moment by moment slopes of the final curve:



The Horizontal Problem, Newton's Cannon

Everybody who looked at the sea knows how ships appear first with their top. In short, the earth is round! Then of course, our falling bullet can go much longer than its simple drop time. The bigger its initial forward speed is, the more this curving of the earth can increase the falling. But, our calculations themselves are all more and more incorrect with the curving of the earth, because the A acceleration vector should point to the center of the earth, so can't be kept as a fix vector. Even more disturbing is the V_0 velocity. If we strictly keep it fix, then it will point away from the earth.



So then, much more extra A acceleration would be necessary to pull back the bullet to the ground. In spite of this, the above picture is truthful in a sense! Namely, if a bullet is shot with a large enough velocity, then it must leave the earth. After all, by Newton's First Law, all launched bullets would keep their velocities if the earth weren't there. So with larger and larger speeds, the earth's effect should be practically eliminated. But then, knowing that large enough speeds make the bullet leave the earth, almost in straight line and knowing that slow enough speeds make the bullet fall to the ground almost on a parabola, we can continually go from one to the other. Decreasing the speed that is enough to leave the earth, clearly the leaving is curved back, while increasing the parabolic drops will make the bullet more and more follow the curving of the earth. So then, the two must meet in a situation where the bullet is exactly shot with a speed, that makes it go around the earth, and comes back at the same height. Now if the "horizontal" speed remains independent even after these curvings, then it will mean that the bullet comes back with the same speed as it was shot! So it goes around again and again! It becomes a moon!
Newton made a picture of this, but instead of the gun, he used a cannon on top of a big mountain. Of course, even this is unrealistic because the highest mountain, the Mount Everest, is still under the atmosphere, so the air would slow down the cannonball eventually. Newton was reluctant to use this picture and only put it into later additions of his book. The reason for this was not the unrealistic feature, rather the many hidden assumptions we mentioned above. The easy transition from earthly falls to the orbiting of the moon is very convincing with the bullet or cannonball picture. We almost feel saying "you see how simple it is". But actually the transition is not simple at all.

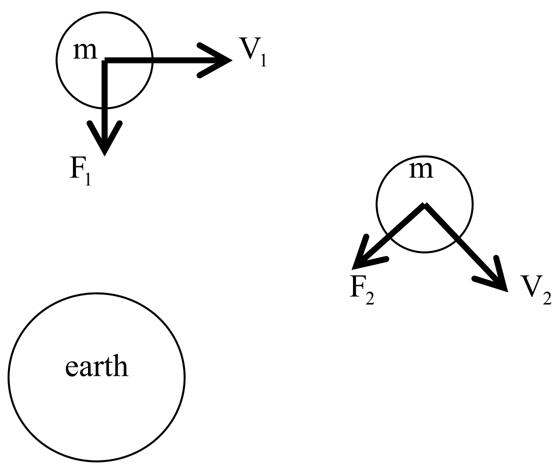
Moon Paradoxes

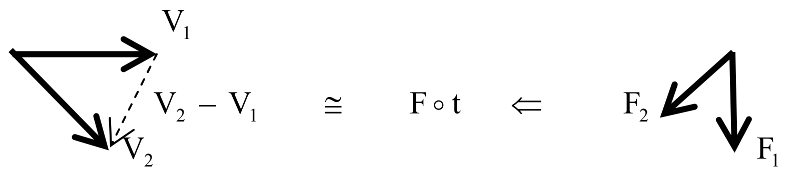
If we accept that weight is not a special force, caused by planets rather a universal force between all masses, which we called gravitation, then the moon is just like an apple attracted by the earth. Then of course the obvious question is, "Why the moon doesn't fall to the earth?". This could be called the first moon paradox, to which of course, the correct answer is that it does fall! But just as an apple can be thrown forward, and thus return to its original position as our bullet or cannonball above, the moon obviously had a proper initial speed already that makes it go round again and again. We can even argue that this "coincidence" of exactly having the right speed is not surprising because other bodies either fall to the earth or leave forever, so we only see the moon that remained in its perfect orbiting.

Then the more astute person, who heard about the laws of common fall and fix acceleration, can reply that, "Why is the apple increasing its speed when it falls, while the moon is keeping the same speed?" This is the second moon paradox.

The answer to this is the transition from speeds to velocities.

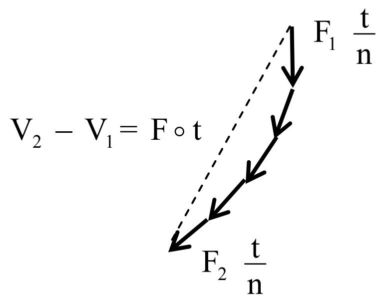
Every F force's $F \circ t$ effect is the $m_2 \ V_2 - m_1 \ V_1$ momentum change, which at fix m mass is $m\,(V_2 - V_1)$, so means a velocity change. If something is falling straight down, then V_1 and V_2 have the same direction, so $m\,(V_2 - V_1)$ can only be the effect if V_1 and V_2 are different in size. The speed must increase! If something is falling not straight down, but with a side speed too, then V is not fix in direction, so $m\,(V_2 - V_1)$ can be the effect, even if V_1 and V_2 have the same length. The speed doesn't have to increase or decrease. The earth's gravitational F force upon the moon in a t time has exactly the $F \circ t$ effect that equals the $m\,(V_2 - V_1)$, with equal long but different directional V_1 and V_2.





Here the $\cong$ symbol means the equality, but without the m factor or equality if we regard the m mass of the moon as the unit of mass.

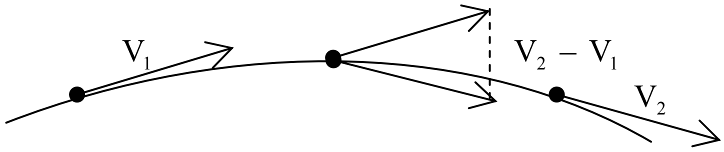
While at the left, $V_2 - V_1$ is obvious from the triangle rule, on the right it's not apparent why F_1 and F_2 leads to this same $F \circ t$. To see this we have to do what $F \circ t$ means, that is add up refined little $F\,\frac{t}{n}$ vectors between F_1 and F_2. The first will be $F_1\,\frac{t}{n}$, so it is actually just a shrinking of F_1. Then, we must add in chain to this a same long, but little bit altered F turned to the left. Again and again, we chain the little vectors together, till the final $F_2\,\frac{t}{n}$ is added. Then, the short cut or sum of this chain will indeed, be the same as $V_2 - V_1$ above:



So everything works out magically! The perpendicularity of F to V guarantees that the constant sized V-s have the same difference as the effect of the constant sized F-s! After this answer a person with deeper attention to details will be not only satisfied, but will truly appreciate the vector meaning of Newton's Second Law too.
And yet, he can think of the Third Law and raise a new third paradox:
By the Third Law, not only the apple is attracted by the earth, but the earth by the apple too. Of course, by the time the apple falls down, the earth could only move a minute distance because of its huge mass. On the other hand, the moon is falling forever, so "Why doesn't the earth fall to the moon?". Obviously, it has to fall continuously, similarly as the moon is orbiting the earth. But how can they orbit each other at the same time? Amazingly, it's simpler than we think, because any orbiting can be looked from the orbiting body and then the other in the center becomes the orbiting one. Indeed, looking from the seat of the merry-go-round, the world is spinning around us. That's nice, but if we look at an even bigger picture, lets say that contains the sun too, then we are in big trouble! The earth orbits the sun. Of course again, we could reverse the roles, and then the sun orbits the earth. But wait a minute, that was exactly what Aristotle and the church claimed. So then the whole change for the sun centered world view was a mere alternative possibility? No, because there is again a wider picture, namely the stars. If the sun is regarded to move around the earth then all the stars have to move around too, and indeed, that's what was assumed. But that required two motions, one daily turn of the whole universe, and one slow yearly change of the axis for the sun. With the reverse view, all the stars and the sun are still and the earth can perform both motions with one single orbiting and spinning around a fix axis. This is clearly infinitely simpler and so we assume to be the real situation. The problem with it was only that the planets didn't fit in, until Kepler's final system. So if we assume that the stars are still then the earth must orbit and then the moon's going around the earth is actually an orbiting around an orbiting point itself. Still, then we can't just reverse and say that the earth orbits the moon too. The final solution is a compromise! They both orbit around a point that is in between them according to their weights. Furthermore then, this center is that orbits the sun. So the earth and the moon both go around the sun in funny loops. Looking from either of them, the other goes around. The "actual", that is sun orbiting center, is inside the earth, so it makes much smaller loops, rather waves, while the moon is running on self crossing cycles.

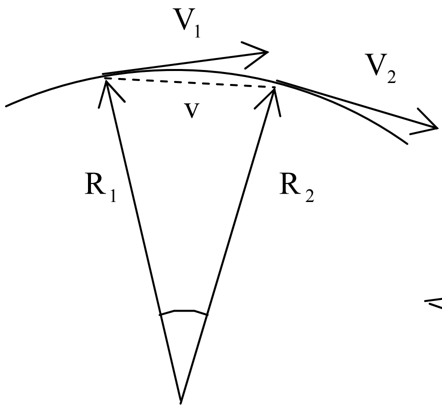
Acceleration Of Orbiting

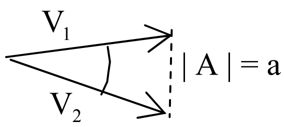
The $F \circ t = m(V_2 - V_1)$ equation being true with fix length F and V is the perfect triumph of vectors as forces and velocities. We also mentioned that $A = \frac{F}{m}$ is an alternate form of the Second Law, if m is fix, but it hides the physical meaning behind the mathematical concept of A. Indeed, F is clearly pointing to the earth, if m is the moon, so then this form claims that the acceleration is also a fix vector towards the earth. Then we can verify this by checking the velocity change rate for a fix long velocity around an orbit:



As we see, $V_2 - V_1$ is not quite pointing towards the center, but as V_1 and V_2 approach each other, the connecting $V_2 - V_1$ is more and more radial. Of course, the length of $V_2 - V_1$ shrinks to 0, but that's okay, because $\frac{V_2 - V_1}{t_2 - t_1}$ will not, rather approach the fix length of $A = \frac{F}{m}$.

We can even obtain this fix $|A|$ length if we look at a bigger picture containing the center of the circle and take two snapshots at a second apart at R_1 and R_2 positions of the r radius.





The perpendicularity of R and V gives that R_1, R_2 and V_1, V_2 enclose the same angles. Thus, the two equal sided triangles with the dotted third sides, above are similar. Also, the one second time choice makes the dotted side between R_1 and R_2, about $|V_1| = v$ long, while between V_1 and V_2, about $|A| = a$ long.

So, $\frac{a}{v} = \frac{v}{r}$ and thus, $a = \frac{v^2}{r}$.

Then, $A = \frac{F}{m}$ means that $a = |A| = \frac{|F|}{m}$, so $\frac{|F|}{m} = \frac{v^2}{r}$.

Kepler's Third Law

If m above is an orbiting planet around the sun, then the circle orbit is invalid. Kepler's basic discovery was, that the orbits are ellipses! In spite of this, most planets orbit very close to a circle, and the differences only show in the observable speed ups and slow downs. In fact, even Kepler's own Third Law, that only regarded the full orbital times, that is the years of the planets, can be obtained from the $a = \frac{v^2}{r}$ circular acceleration. But a new twist is still important!

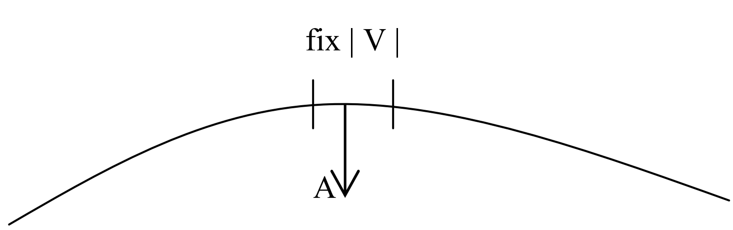
If m is a planet above, then F is the gravitational pull of the sun. It always points towards the sun and if we disregard this direction and only care about its size, then this $|F|$ is the weight of the planet by the sun.

Just as we regarded the weights of objects on the earth to be proportional with their masses, it seems natural that the weights by the sun are too. So, $\frac{|F|}{m}$ should be a constant, that is the same for all planets. This is incorrect! The big difference is now

that the planets are in very different distances from the sun. On the earth's surface, the objects are practically all the earth's radius away from the center. Even going up to the Mount Everest is only a minute change in distance from the earth as a whole.
It's quite natural that going away from a mass, the gravitation must decrease. But how fast! We'll return to this question, but now only look at it from the point of Kepler's Third Law. First of all, we can realize from the right side of the $\frac{|F|}{m} = \frac{v^2}{r}$ equation too that it can't be constant for the different planets. Indeed, if $\frac{v^2}{r}$ were constant, it would mean that the planets on larger orbits had larger speeds too. This completely contradicts Kepler's Second Law, which described how a planet, when gets closer to the sun, speeds up. Of course, that law is talking about one planet, while the third law is talking about the different planets. Still, it would be very strange that one planet becomes faster as it gets closer to the sun, while the planets closer to the sun were slower than the ones further away. In fact, Kepler's Third Law exactly tells the opposite, namely how much slower the outer planets are. Instead of $\frac{v^2}{r}$, exactly the opposite, that is not the ratio of v^2 and r, rather their product is constant.
So, $v^2 r$ = constant, is basically Kepler's Third Law, but he used the T orbiting times, that is years of the planets instead of their v speeds. This is logical, because the planets are going on ellipses and thus, change their speeds. In fact, that was the whole point of introducing the elliptical orbits. But now, if we simplify the orbits as circles, and the changing speeds as fix v speeds then this can be obtained as: $\frac{2 r \pi}{T}$.
Indeed, $2 r \pi$ is the length of the orbit and T the time required to travel through. Writing this into v, we get $\left(\frac{2 r \pi}{T}\right)^2 r = \frac{4 r^2 \pi^2}{T^2} r = \frac{4 r^3 \pi^2}{T^2}$ = constant and since $4 \pi^2$ is constant, thus $\frac{r^3}{T^2}$ = constant. Remember, the word constant here means the same for every planet. This is the form Kepler stated his third law.
From $\frac{|F|}{m} = \frac{v^2}{r}$ multiplying both sides with r^2, we get $\frac{|F|}{m} r^2 = v^2 r$. The right side being constant is Kepler's Third Law and so the left side must be constant too. So, we found out how the F gravitational force of the sun must decrease with the distance, namely it has to drop in proportion to the squared distance. And of course, that will be exactly what Newton claimed and so in reverse from that, Kepler's Third Law follows too.

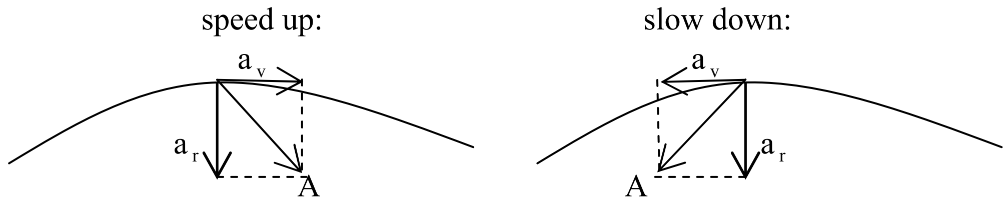
Circular Approximations Of Arbitrary Paths

The fix length of A above was obvious because, a circle is the same all around!
If the body moves on an arbitrary path, but the length of V, that is the $|V|$ speed is remaining the same on a part of the path, then still the $\frac{V_2 - V_1}{t_2 - t_1}$ limit again will be "radial", even though there is no fix radius. Instead, simply meaning that it will be perpendicular to the path and pointing inward the curving.



In fact, even the "radius" can be generalized as the circle's that fits on the part of the path. And then, even the $a = |A|$ length can be calculated from this radius, the same way as for circular orbit, that is $a = \frac{v^2}{r}$.

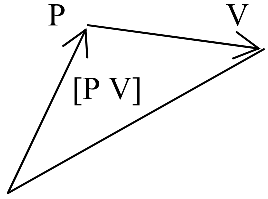
Finally, if the body is not moving with fix speed, then its acceleration can be split as the $a_r = \frac{v^2}{r}$ radial component perpendicular to the path and an additional a_v speed up or slow down component, lying in the line of the velocity. At speed up a_v points forward, while the slow down backwards. The sum of these two components of course, is the real A acceleration and thus, $\frac{F}{m}$.



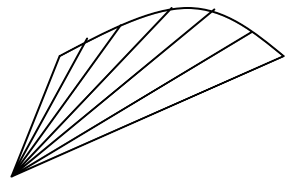
This general acceleration picture even works for motions in straight lines.
Then the curvature is infinite, that is $r = \infty$ and thus, $a_r = \frac{v^2}{r} = \frac{v^2}{\infty} = 0$.

Aerial Speed, Kepler's Second Law

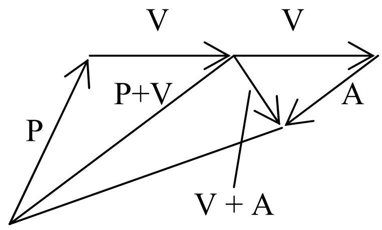
A completely different generalization of the circular orbiting can also be obtained! Instead of the perpendicularity of F and A to the path, we can stress that they pointed towards a central body. So then we can ask, if there are other motions than circular orbitings with such central force and acceleration. The answer is the obvious yes, exactly from the physical meaning. Indeed, if we spin a rock around on a rope, then we are the center and the rock orbits in the circle around us. But, if we pull in the rope or let it out again, the force always remains through the rope and thus towards us. So we get infinite varieties of such central motions. Amazingly, we can give an other description of all these motions, namely as Kepler's Second Law. In other words, the swept areas of the rope are the same under the same times. But then this, will mean that Kepler definitely didn't discover God's secret law of the planets, because the same law is true for the much wider range of central motions, including spiraling of a stone when a child pulls in the rope. In fact, we will prove something even more general which shows why in the "equal areas under equal times", the choice of time is immaterial. Instead of the swept area, we can look at the area of the triangle between the rope and the velocity:



Firstly we claim that the actual swept areas can be easily obtained from these [P V] triangle areas, namely as their effect, that is $[P\,V] \circ t$. Indeed, these triangle areas multiplied with $\frac{t}{n}$ time intervals, we'll make tiny triangle areas that add up to the actual swept areas:



Now all we have to show is that [P V] is fix, because then [P V]$\circ$t = [P V] t, so indeed the swept area will be proportional to the time, in particular under same times will be equal.
Lets look at a P location at t time with velocity V and then a little time, say a second, later. The new location is approximately P + V and the new velocity is approximately V + A.



But, [P V] = [P+V V] = [P+V V +A].
Indeed, all the three triangles have the same P+V base and the same height if A is parallel with P+V.

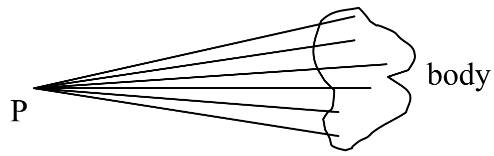
Kepler Paradox

When we guessed Kepler's Third Law in the $v^2 r$ = constant form, we merely changed $\frac{v^2}{r}$ to $v^2 r$ because we knew that larger orbits must mean slower speeds.
On the other hand, above we realized that Kepler's Second Law is true for all central forces and actually is caused by [P V] = constant.
If we imagine our stone on a rope, orbited on a larger circle and later pulled in and orbited on a smaller circle, then on the two circles, the speeds will be fix and perpendicular to the radius. Thus, on these two circles, [P V] = r v, so this must be constant. Now if we imagine that an outer planet, say Pluto is pulled in similarly to become an inner planet, say Mercury, then the same should be true!
Thus, r v = constant, that is the same for Pluto and Mercury. But, by Kepler's Third Law, $v^2 r$ is constant, not r v. Obviously the two can not be true at the same time, because the first is v times of the other.
So it seems as if Kepler's Second and Third Law would be in contradiction!
The error we made was to apply the law of central force in a false way. It applies to one body's motion as it changes its path and not to all possible motions. Indeed, even the same rock can be orbited on the same circle with different speeds. So r v won't be the same for these two cases simply because r is the same, but v is not. And indeed, to speed up a rock on the same orbit, we have to move our hands out of the center, make jerky movements and thus, use non central force. Of course, the sun has always been in the center, so the planets seemingly should be all under the rule of central force. And indeed, they are under central forces through their separate motions. But each of them came separately and have their own separate central force. So, just as we could spin different rocks from one center, the planets are moved by separate central forces. Just as we can pull in and let out the different rocks and they will all obey the r v = constant law separately, the planets each obey this same law under their elliptical orbit changes. But while, the differently orbited rocks can have completely unrelated central forces, the planets all have their central force determined

by gravitation. And thus, while the different rocks can have totally unrelated speeds and distances, the planets obey Kepler's Third Law that is, $v^2 r$ = constant, meaning the same for every planet.

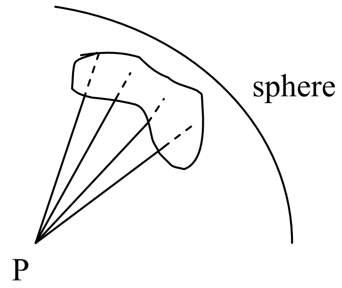
"Radiation" Of Gravitation

If we assume that gravitation is spreading out from all masses in all directions of the space, then we can obtain the exact law of this force by "simple logic". As always, with such "derivations", we have to do some hidden assumptions too. Right to start with, the expression in "all directions", must be meant in a tricky way, namely that all directions are equally possible, but not literally as towards all lines. Indeed, the total of lines going out from a point is infinite and so no matter how small gravitation is spreading out into each line direction, the total would be infinite. So, into one particular direction the portion of gravitation that spreads out must be zero. But then, we could argue that if one line's portion is zero, then a whole bundle of lines that connect between the points of two bodies is also zero in total. To overcome this dead end argument, we can accept that the jump from actual physical bundles of lines to individual mathematical lines is simply beyond our imagination. So should we give up the concept of line? No! Instead, we can regard a particular line as an undetermined one physically. Then it can possess a small gravitation but it's not its own, rather to be shared with the surrounding line directions. Then we can very well imagine marked lines that carry the gravitation but when we use these lines, they must be sharing this with their surrounding lines. So their choice is arbitrary, we could pick other marked ones and only the amount of marked lines in a bundle really counts. The simplest bundle of lines are ones that originate from one single point. So, we can regard our body as a single point and the gravitation radiating from it as marked lines towards all directions. We already know that the gravitational force between two bodies is proportional with the two masses. But now, regarding gravitation as radiating from each mass, we must separate this two sided proportionality and in a sense return to the more primitive one sided concept of weight. So, the marked lines that spread out from a point mass and represent its gravitation, can be simply proportional to the mass of the body. So, if at a P point, a more massive body is imagined, then simply we imagine more marked lines spreading out in all directions.
But, the dependence of the gravitational force from the other body can be easily fit into this picture. All we have to do is regard the other body not as a point, rather an actual physical one! Then, it will encounter a certain amount of the marked lines radiating from P:

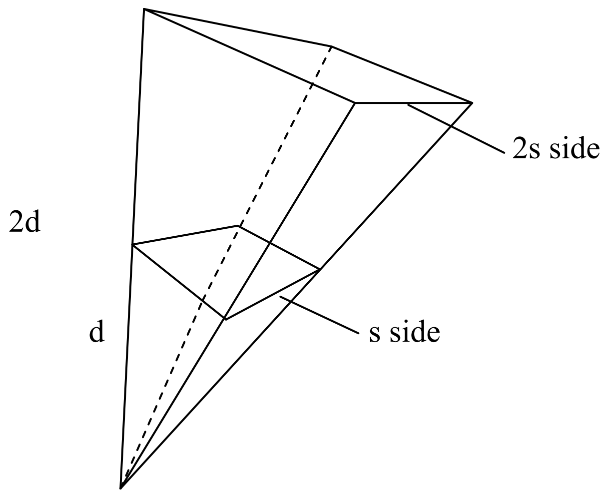


To get the actual gravitational force, we simply have to multiply the number of these encountered lines with the mass of the encountering body. In fact, we can even imagine that the physical body above has more and less massive parts at different places and then we can multiply the number of lines only at those places with their separate masses. Thus, we can have different gravitational force at different parts of the body. But first, we have to ignore this possibility and rather assume that the body is homogeneous, that is the same at all places, in order to find out how for a fix body, the gravitation depends on the distance. For homogeneous body we only have to multiply the number of crossed lines with the total mass of the body, so we only have to find out how the number of these crossed lines depends on the distance from P. Of course, the body consists of different points and each are at different distance from P.

One way to get a single d distance would be to regard some kind of average of all the distances of the different points of the body. Or, we can assume the body to be far enough, so that the different points are practically all the same d away from P. Or, we can even go one step further and assume that d is actually the same for all points. This means that the body must be imagined as a patch on a sphere with d radius:



Unfortunately, this idealization is contradicting our original goal to find out the dependence of crossed lines for a fix body, because a patch on one sphere could not be on a different sphere too. Of course, what we mean by a body is the same shape and size and luckily, the shape is unimportant too, because all the parts count equally. So, only the size, namely the area that determines the number of crossed lines.
Thus, the final question is, how a fix area's crossing lines depend on the radius of the sphere. The solution is quite easy! The total of crossed lines is the same for all full spheres. The lines are equally distributed in all directions and thus, on all spheres as crossing points too. Then, the number of crossings on an area is the same proportion of the total lines as the area is of the full surface of the sphere. So, if we have a fix area, then the proportion of crossings is decreasing with d exactly as the surface of the sphere is increasing with its d radius. This surface can be approximated by little squares and each side is increasing proportionally with d. For example, at doubling the radius:



So, the area of a square will be increasing four times.
Or in general, the surface increases with d^2.
Now it's easy to tell the gravitational force between two m_1 and m_2 masses that are d distance away: $F = \gamma \frac{m_1 m_2}{d^2}$.
The γ factor contains three assumed proportionalities together. Firstly, how many lines originate from the m_1 point mass. Secondly, how much a sphere's surface is proportional to its radius' square. And finally, how much the second mass m_2 is increasing the force.

Density

Similar "field lines" as the gravitational ones that we used above, can be defined for other forces. Best example is, the magnetic ones we see in diagrams, to show how the compasses are aligned. Sometimes they even spread iron powder on a paper and when the magnet is placed under, we can actually see the connecting lines between the poles. This effect is a trick, that has its detailed explanation through the combining of the magnetized iron powder.
But, there is a much better abstraction than the field lines and we already used it!
Indeed, just as in space to narrow down to one point is a problem, in time to narrow down to one instant is problematic too. When we introduced the moment by moment speed or velocity, we simply jumped through this whole problem and relied on our instinctive common sense to give the right meaning. Newton's First Law helped a lot, because by that, the velocity of a body at a moment means the fix velocity for all future time, if all influences would be disappearing. This moment by moment changing velocity was also shown mathematically as the limit of the average speeds under smaller and smaller time intervals that narrow down to moments. Indeed, as we saw $\frac{P_2 - P_1}{t_2 - t_1}$ will approach the V velocity at the t moment if t_1 and t_2 approach t. This is an actual vector, even though both the numerator and the denominator approach zero separately. The moment by moment velocities have their physical effect by determining where the body will be. And indeed, we even used the word "effect" to calculate how this determination is happening. Just as a fix v speed under a t time interval causes a $v\,t$ distanced motion, we can define the $v \circ t$ or $V \circ t$ partitioned products. This was merely cutting the t interval into small pieces, then in each regarding v or V fix and thus multiplying with the little times and then adding all these together. This, in mathematics is called integration and there using the mathematical definition of speed, it becomes a theorem that the integration of the speed gives back the distance. Or in general, the integration of any change rate of a quantity gives back the change of the quantity. I even claimed that this so called Newton-Leibniz rule is used by all humans, maybe even animals, to estimate changes from our memories.
To call the partitioned products with t time intervals involved as effect is very natural. For example, the effect of the speed is the traveled distance, the effect of the velocity is the displacement and the effect of the acceleration is the change of speed of velocity, and so on. The same ideas that I repeated above can be used for space instead of time. In one sense, time is more complicated than space, because time seems to have a direction. On the other hand, space has dimensions. If we regard the full three dimensional space, then what would correspond to a time interval is a volume. Unlike a time interval, a volume has shape, so it's much more complicated. But many times this shape is unimportant and then, just as from time intervals we jumped to individual moments, we can approach points from volumes. Then, the change rate will correspond to the concept that we usually call density. For example, just as the most important change rate, the speed tells how much the traveled distance is around a single moment, the most important density, the density of mass, which is usually just simply called density, tells how much the mass is around a single point. Just as the speed had to be multiplied with a time interval to get the actual distance, the density has to be multiplied with a volume to get the actual mass. But just as instead of speed, we used velocity as vector, density is also just as meaningful for vectors. So, instead of the gravitational lines, we could have introduced gravitational density vectors, which when multiplied with a volume give the gravitational pull from a point. Or, if multiplied with not just the volume, but with the mass, gives the actual gravitational force. The whole idea of the partitioned product can be repeated. Instead of a t time interval we have to cut a u volume into pieces. This again is called

integration in mathematics, and could be called the effect of a quantity over a volume. The end result is the same, namely that these partitioned products of the densities give back the full quantities over the u volume. For example, the effect of the δ density (of the mass) over a u is the mass itself in u. In equation: $\delta \circ u = m$. Similarly, the effect of the gravitational density is the gravitational force, and so on.

Moment Paradoxes, Zero Paradoxes

Today we regard time as a line, in fact we even use the expression, "point of time", for the moment or instant. The distinction between time and space is still an unsolved problem, and the first contradictions regarding time with space were raised by the greek sophistic philosophers Parmenides and Zeno. Before we even react to these, we should emphasize that the most amazing is not how they tried to show that time is different, rather how they were already in full acceptance of the points of space.

When an arrow flies it is at one place at every moment and yet, it is not at rest, so it can't be at one place ever. Today, we would say that there is no contradiction here, simply the word "ever" was misused. Just as space contains points, time does too as moments. So every moment is a different point in time, and just as time flies, the arrow too. To be in rest, means that for a small time interval, the object is at a fix point in space. For the arrow, this doesn't happen. It is at one point, at every moment, but not under any time interval.

Achilles is 100 times faster than a turtle, so gives 100 metres head start to the turtle, in other words, starts 100 metres behind the turtle. He probably will still beat the turtle, but Zeno says he can't even catch up with it. Indeed, lets see where the turtle is when Achilles gets to the point where the turtle started. Clearly, under this time, the turtle will move one hundredth of what Achilles did, so 1 metre. Now lets see where the turtle will be when Achilles finishes this 1 metre. It must be again one hundredth of it ahead, which is 1 centimetre. Then again, we can check when Achilles reaches this points but the turtle will be again ahead. So, the turtle will always be ahead.

Just as in the arrow paradox, the word "ever", here the "always" is misused. It is indeed, true that we produced an infinite number of moments when the turtle is ahead of Achilles, but that doesn't mean forever or always. If the rain starts to fall now, then there were infinite many times before, when it was not raining. Indeed, just a minute ago it wasn't raining. Half a minute ago, it wasn't raining either. A third of a minute ago, neither. And so on, we can go back to smaller and smaller times, when it was still not raining.

As we see, by regarding time from points just as space, the contradictions seem to disappear. But this is an illusion because, we didn't tell how the points of space should be defined exactly. So only the sharp distinction between space as points and time as small intervals disappear, while the common problems of points themselves still remain. These problems only surfaced when the concept of sets were introduced. Then, the question was finally asked, how the points are located on a line.

When Euclid regarded the crossing of lines as points, then clearly this suggested the idea that the lines are sets of points. But until any collection of points could be raised in general, the special point sets, namely the lines, circles and planes, were left unresolved too. An other factor of this long ignorance was that geometrical concepts were approached algebraically, namely distances became numbers. So when calculus was created, it was formulated for numbers. It only turned out later that actually it was dealing with points. Topology became the wider field, but only after Set Theory.

Even today, the newer paradoxes of points can be formulated as paradoxes about the zero number. This shows that we couldn't penetrate deep enough to the heart of the problems. Still, I would like to mention a few of these paradoxes:

The simplest problem of choosing a point is that it is actually impossible! Indeed, to pick one particular point from the infinite many that lie on a distance, area or volume, is probabilistically impossible. If a small distance is one millionth of a bigger, then still after every millions of picks, we can pick one from the smaller one. In other words, any two distance is comparable. But one single point is incomparably smaller than any small distance. And yet, we have to accept that we can pick a point. We can throw an imaginary mathematical dart on an imaginary mathematical dart board and it will land at one single point. So then, we might say that somehow, time must be involved. Namely, if we predict a point on the board, it is impossible that we throw exactly there, but every throw will land in an unpredicted point. This paradox is still ignored in mathematics, because others appeared without even time.
With the concept of arbitrary point sets, the old obvious point sets, like a full distance, circle, disc or ball, were able to be separated into strange components. Now if we cut, our imaginary dart board disc into no matter how strange components, it will be still true that every throw must land in one of the components. This would give the impression that each of the components has a certain chance according to its size. But what if, we can separate the dart board into infinite many components, and in addition we can prove that the components are identical, so must have the same chance. Clearly, then each component can only have zero chance, because no matter how small other chance one had, the infinite many together would make infinite total chance for the whole board, even though we know that it must be 1. The most obvious such equal distribution of the dart board is to regard each point of it as a component. The chance of one point is indeed 0, but the infinite many together still gives 1. This in itself is strange and resembles our earlier arguments about the gravitational lines and partitioned products and densities as solutions. Indeed, we can say that though the points have 0 probability, they have their non zero probability densities, which when multiplied with the little areas give the actual probabilities. We can even see that in the center the probability density is the highest, even though usually this has the highest score. But now, if somebody can split the dart board disc into a sequence of $S_1 , S_2 , \ldots$ sets, that are identical, then a whole new contradiction appears. First we might say, that nothing changed, again each will have 0 chance, and together they give 1. But now we can add up finite many of our sets and still would get only 0 chance. For example, lets start with S_1 as a first set. Then add S_2 to it and regard this $S_1 + S_2$ as our second one. Then add S_3 to it and regard it as a third, and so on. Thus, we obtained a sequence of bigger and bigger sets that approaches the full disc. And yet, each of the approaching sets has 0 chance, while the final total has 1. Obviously, such components must be very weird! But, even weirder ones were created, that can be moved into themselves. Any sequence can be shifted into itself, for example, the sequence, $1 , 2 , 3 , 4 , 5 , \ldots$ can be shifted ahead to start from $4 , 5 , 6 , \ldots$ This way of course, we only lost three elements, namely $1 , 2 , 3$. We can also see that a sequence, like $1 , 2 , 3 , 4 , 5 , \ldots$ is even similar to its own subsequences, like $1 , 3 , 5 , \ldots$ In short, the odd numbers are just as many as all the numbers together. This weird thought occurred to Galileo already when he formulated his consecutive falling times, with the odd numbers. Probably, these wonderings lead him astray from the more fruitful discovery of the time squared falling distance. Anyway, we have to realize that this similarity of a sequence to its every second members, is not a shift anymore, so it couldn't be used for point sets. If however, we roll up the numbers on the perimeter of a circle, and use twists instead of shifts, then a whole new door opens for rigidly moving point sets. The final result was that a sphere can be split into just a few point sets so that they can be again reassembled, but obtaining two spheres of the same size. Then of course, layer by layer, a whole ball or as they usually put it, an orange can be doubled. Again, just like above, with our dart board components, it's the number of components that makes it really surprising. There it was a sequence that made it strange, and here the

even more surprising fact that it's only a few, in other words finite. We can easily split a sphere into components, so that we can increase the sphere and keep the components, namely regard a sphere as the combined single points on it. Then, from the center, we can simply project it onto a bigger sphere. "Amazingly", every point will be transferred to a point of the bigger one. But this, surprise itself shows that infinites are tricky and many unexpected results can happen.

Finally, a physical paradox of the zero size should be mentioned. Not surprisingly, it relates to vectors. When a vector becomes 0, we might think that just as it lost its length, it must lose its direction too. But this is not obvious! If we drop an object, it starts with 0 speed, but if we trace back its motion, we can define a limit as its initial direction, which obviously points downwards. Or, more physically the surrounding conditions define that it will move downwards. Forces are even more tricky! They can add up to a total 0, and yet have different meanings. For example, the book that lies on the table has the total force upon it, being 0 and indeed, is in a stable rest. But actually, this is only true because, the table itself has friction and doesn't allow the book to slide sideways. A boat is still staying in rest but much smaller force can conquer the friction of the water. If a ball is placed in an open hemisphere, like a wok pot, it will stay on the bottom, totally stable. Here the total forces are not only 0, but all changes in the position of the ball would encounter forces against. Finally, if the ball is placed on top of a hemisphere, then the total forces might be 0, but the ball would probably roll down randomly somewhere, because the slightest displacement will cause moving forces. So, as we see the 0 total forces still can hide other conditions. Our present vector concepts don't deal with these 0 possibilities.

Finally, a personal note to the zero problems. When I was in high school, I thought I discovered a new number system where zeroes are differentiated and can be used instead of limits. This was achieved by using sets placed into each point of the line. I was so enthusiastic about my result, that I sent a short description to some Nobel prize winner physicists. Only Heisenberg replied, but he did within three days. Later, when I learnt English, I realized that Abraham Robinson already worked out in full detail "my idea" and called it Non Standard Analysis. Most amazingly, he also felt that it somehow could relate to Quantum Mechanics and recommended it to physicists, in an epilogue. Unfortunately, it is still unused in physics, so probably some deeper discovery of the zero problem is necessary to revolutionize physics.

<u>The Ball Problem</u>

In the $F = \gamma \frac{m_1 m_2}{d^2}$ gravitational law, d is the distance of the m_1 m_2 masses.

If the masses are far, compared to their sizes, then their sizes are irrelevant for d.

If however, one of the masses is big and the two are close to each other, then it's a real dilemma what d should mean. This is the case with the earth and the apple.

Obviously, d can't be the closest distance, like the apple's height from the ground because most of the mass of the earth is much further than this. Newton solved this problem by distributing the bigger mass, like the earth into small pieces and combining their individual gravitational effects. This is not an easy job because every piece will have a gravitational effect reversely proportional to its d^2. And yet, the end result is the simplest possible, namely that the whole big mass can be concentrated in its center. In short, for a planet we have to use its radius as d.

So, the gravitational force on the surface of a planet with M mass and r radius, is

$$F = \gamma \frac{m M}{r^2}.$$

Distantial Gravitational Tension

The $F = \gamma \frac{m_1 m_2}{d^2}$ gravitational law means that the force is decreasing quite strongly with the distance. That's why outside the solar system the sun's gravitation is practically nothing. On the other hand, on one planet, like on earth, we regarded the weight independent of the height because, even the highest mountains are mere fractions of the planet's radius. Yet, if we want to be totally exact, then even for one apple, we have to realize that the lower part of the apple is closer to the earth and thus, has a bigger gravitational effect than the upper part. This difference in force, means a tension. Of course, the apple's rigidity has to cope with the whole force itself, so this tiny extra tension doesn't make any difference. When the apple drops, the tension from the gravitation and its counter forces disappears and only this gravitational tension remains, but it won't have any effect on the apple. If we imagine a softer body like a balloon filled with water being dropped, then we could even imagine how this tension would shape the ball into a elongated ellipsoid. This of course, this again could never be measured, especially under the short time it falls to the ground. On the other hand, we might argue that if we launch our water balloon like Newton's cannon ball, just with the right speed, then it will become an orbiting satellite and thus, it has all the time to acquire the prolonged shape caused by its tension.

Directional Gravitational Tension

The apple endures an other minute but quite different tension caused by the earth's gravitation too. Namely, all the particles of the apple are actually attracted by the combined forces of the earth particles, which as we saw can be summed into the center of the earth with the full mass of the earth being there. So all apple particles experience a total force directing to this single point. This means a not perfectly parallel accelerating force that wants to squeeze the apple together towards its direction of falling. So it has the same effect as the above distantial tension.
In a sense it helps that to form an elongated ellipsoid for a water balloon.
Of course, the theoretical mass concentration applies to the apple too, so the same tensions are caused by the apple to the earth's points. Thus the true root of the tensions are caused by all individual parts of the apple and earth acting on their own and the above arguments are merely calculating the final tension on one of them.

Free Fall

When an object obeys the law of gravity, it is in free fall. This includes Newton's cannon ball, that is orbiting satellites, moons, planets too. But why should we give this special name for an object simply obeying a particular force. Obviously gravity is a very special force because it is the only absolutely universal force. This means that if an object is experiencing gravity, all particles of it automatically obey it too. So the parts don't have to transfer this force to each other. Unlike an accelerating car that accelerates us through the seats and thus we feel being accelerated, in free fall, a car and everything in it is accelerating together.
This resembles Newton's First Law about the forceless translation, that is fix speed being kept for ever. That law already had the same hidden agenda. It is true for everything, thus if a whole object or objects together are in same translation then they relative to each other don't have to do anything, so are unaware of the translation.
We all know that inside fast trains we can pour drinks, play golf. Most of the time even in airplanes.

The more intriguing questions is how far this unfelt speed can be increased and relative to what should it could be measured. This leads to Relativity. And then this gravitational free fall becomes an even more amazing possibility. Namely, an extension of Newton's First Law. So then gravitation is not a force rather a new environment where translations that is fix speed motions are now curved both conventionally as curved lines toward the pulls but also in time, so accelerations are merely time curved lines. The point is that all matter obeys the same curved environment. So accelerational laws can be interpreted as new laws of gravity.
This was exactly the idea of General Relativity.
But even without wondering toward the translational, that is Special Relativity ideas, the common free fall of an environment represents some paradoxes that the early students of Newton's Mechanics completely missed.
Jules Verne described in his "Journey To The Moon" with quite vivid detail the insane idea that in-between the moon and earth at the right point their gravitation cancels and that's when the travelers experience weightlessness.
Of course the whole earth saw how in the orbiting space capsules the astronauts are weightless all the time. They are free falling with the capsule.
If an elevator drops, the same way the people in it will float. To shoot films in weightlessness, usually a whole airplane is free falling for minutes and then they turn back the engines.
These free falling systems are now called being in artificial weightlessness.
The two tiny tensional forces of course mean that these artificial weightless situations are not perfectly identical to true weightlessness. Most surprisingly General Relativity regards these free falling situations as identical or even better realizations of true weightlessness. The tiny forces are inhomogenities of the space time curvings. But following the space time is forceless. Quite oppositely, the table that stops a book from free fall is altering the forceless motion and achieves this through a force just as an acceleration is achieved from translation. Rigidity doesn't allow perfect free falling of the parts and thus means again tiny forces.

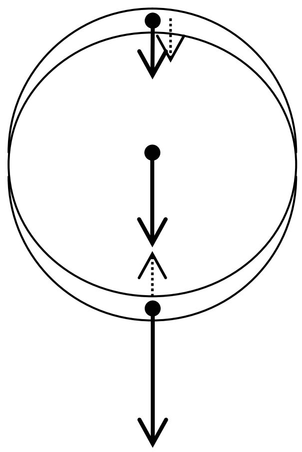
Tides

Tides are the reality of the earlier mentioned two seemingly minute gravitational tensions at planetary sizes. So, the planets and moons are all giant water balloons that by the two tensions try to take shapes of elongated ellipsoids. Of course the solid crust is much less flexible than the oceans around, or the volcanic magma inside. The more important difference from a water balloon is that, here it's not a rubber bag rather the body's own gravitation that keeps everything together.
So, tides not only mean ocean tides because a planetary body itself is never perfectly rigid either. Our moon for example is very rigid without any oceans and with hardly any magma. And still quite amazingly, its feature of facing the same side to us was caused by the tides on it due to the earth. The most well known fact of course is the opposite, the moon causing the ocean tides on earth. Usually they also mention that a similar but smaller tide effect is caused by the sun.
The earth's rotation has no role in the existence of tides but obviously the daily occurrences of the two tides is due to the earth rotating once every day. So the two bulges of water have to actually travel against the earth's rotation. This of course is not actual motion of that water, rather different water is rising up continually. Of course the continents don't allow a perfect double bulge and the continually changing bulge itself works against the rotation. Inside, the volcanic magma is bulging only in density but it still presents an even stronger resistance against a revolution, explaining the mentioned slow down of the moon. The earth will also slow down to the "ideal" rotation of facing the same side towards the moon. Then tides as daily occurrences

would disappear. Of course, all this will never happen perfectly because as we said the sun is causing tides too. That would want to slow down the earth's rotation to face the sun with a same side and not to have a tilt.
So, the rotations are mixed up in the appearances of tides as alternating water levels and also as actual interferences. But the crucial point is to emphasize again and again that the existence of tides boils down to the two bulges of a soft matter being in gravitational field and these are caused by the gravitational tensions, not the pull.
The pull itself "could" only explain a bulge toward the external gravity source.
But we had to use quotation mark, because it is a false explanation that completely ignores the whole Newton's cannon vision or orbiting as reality. The correct vision is to realize that the motions are not happening in the directions of forces. Newton's second Law in vector form means that the earth is orbiting the sun due to the sun's attraction, thus perpendicularly to the force. The oceans as part of the earth do the same. They already obey the sun's attraction by orbiting, so there is no extra pulling off, away from the earth. Indeed, by that token the people should feel an attraction towards the sun but we don't. Or if you wish, we "feel" the sun's attraction, but that's why we travel with the earth and thus strangely we still don't feel this pull in our contacts with the earth. So, the concept of free fall is always hiding behind the true understanding of Newton's Second Law. This is surprising, because seemingly the Second Law is more universal than gravitation.
This is why I went from the gravitational tensions to the free fall and even included the lengthy detour towards General Relativity. In a sense, it was didactically faulty because the tension is a simpler static force. Any object in gravity is deformed by the two gravitational tensions. To put it even plainer, if giant hands would grab all planets and moons, the tides were still there. Newton's Second Law and the orbitings are not necessary. But as I said, the appearances and interactions involve the orbitings and we want to envision it anyway in the full picture. So in the end it was better to wait with the Tides after the free fall. But there is a grander picture anyway!
First of all, the Tides represent a crucial didactical battle ground.
The insane false explanations are flooding the textbooks and the internet.
Since the detailed equations of the actual tides are quite difficult, this allowed the excuse of "we only gave a simple explanation". But behind these "simplifications" we can see the total lack of understanding the essence of Newton's new vision.
But we have to be truthful and admit that Newton and the text books of his mechanic didn't emphasize these crucial new views either.
So the bigger evil is Formalism as always.
In fact, in a strange way, the more oversimplified explanations have more truth in them than the overcomplicated formal garbage.
For example, the simplest explanation of the double bulge is first to use the above mentioned false one, claiming a bulge toward the source by pull, and then as second false step claim that the whole earth is also pulled away from the oceans of the other side. This view is at least static, so it could be seen as a simplification of the distantial tension. Of course still falsely, because it misses the very distance dependence.
The reason that a much worse, more complicated version exists too, is this:
Above, I explained clearly the falsity of the "pulling away" with the sun as center.
Indeed, the earth is clearly orbiting the sun, so the pulling away of the oceans is quite easy to see as false. Unfortunately, for the bigger moon tide effect this view is false itself. The earth doesn't orbit the moon rather in reverse, or if we want to pretend to be more exact then they both orbit the mass center somewhere inside the earth. But this whole line is crazy because that view with the sun was merely to see that the oceans are part of the earth and the gravitational pulls don't warrant anything.
So, to go into the new motions again should be about that and not about the tides.
The tides are results of tension, regardless whether the source is the sun or the moon.
So, the whole over complication of the motions is a cover up not to see the tension.

Could it be that regarding the orbitings from the mass center, the tension becomes part of the motions? Yes it could be, but all these descriptions cross over at a point into the "pulling away" vision. They simply muddy the waters to pull the rabbit out of the hat. But the rabbit was never in the hat. No "pulling" is necessary.
So, summing up the whole affair, the tides which have nothing to do with the motions, became a revealing of how misunderstood these motions by forces are.
But now comes a first surprise. Above, I said that regarding all motions properly, the tensions could become part of that picture. Indeed they have to, because in the end if all parts obey the Newton Laws then that is the reality. But there is a crucial twist.
The common view that if we know the forces then we know how things move is not correct. The Newton Laws are not a simple axiomatic system. The realities of motions as geometrical assumptions sometimes create the forces that will obey the Newton Laws. Then these forces imply more detailed geometry. This is actually a splintered imperfect form of General Relativity already present in classical mechanics.
So now I show this motional explanation of the tides for the simple sun centered scenario, that is the earth orbiting with its oceans:

If something is orbiting closer to a gravitational center, then it has a bigger gravitational force, so it must move faster too. For example, the side of the earth towards the sun wants to orbit faster, while the opposite side wants to orbit slower.
Of course, rigidity and the gravitation of the earth, keeps all parts to orbit together with the center.
The waters can still move easily and tend to create a situation where the total forces correspond to the orbits. The two bulges are doing exactly this.
Indeed, both will cause extra gravitational forces by the earth towards its center.
So the bulge on the opposite side to the sun will add a force to the too small gravity of the sun. On the side closer to the sun, the extra gravity of the earth will subtract from the suns too big gravity. So the two bulges will have same total accelerating forces as if they had been in the center:



Full arrows are gravitational pulls of the sun on a unit of mass. Dotted arrows are the extra pulls of the earth's gravity. The totals on the two bulges are the same as if they had been at the center, so all parts can move together by the Second Law.
So, the original and simple static tension became an enforced tension at once resolved too, by the internal gravity of the earth.
This makes us wonder if the original didactical simplicity was itself false or not.
No it wasn't. We can go to actual motions with water balloons too.
But all this shows that Physics is not Mathematics. In math, Formalism can abuse the subject by splintering it into mere derivations.
Didactical correctness still sneaks in and grabs by the stomach anybody who is honest. So, only with subjectivity involved can we go towards a deeper objectivity.

To put it quite abstractly in philosophical language:
Didactics goes toward dialectics.
The biggest dialectics of nature or the universe is that in one respect only mathematics exists while in the other mathematics doesn't exist at all.
The concrete social contradiction that schools don't teach physics at all but require abstract math on its own, is a conspiracy of the devil to completely block out real math and real physics. Any sane educator would agree that integrated science, containing math is the only logical approach. Instead, math is totally separated from even physics. The root of all this of course is the obsession with derivations. And most shockingly with false derivations.
Didactical correctness, so smart honesty is our only guiding torch in this totally distorted, false and lying age.
When I read Richard Feynman's manifesto about the craziness of physics education, it grabbed me on an other level too. My older brother Peter, had to leave the University of Electrical Engineering "purely" because he couldn't cope with the insane abstract pure math required there. He still did what he loved and knew very well but for less money. In a sense that's what it's all about anyway, money. But not is a simplistic way. Peter's textbooks were written by the wife of the professor of Analysis in my Uni, Akos Csaszar. She wanted to show how sophisticated she was.
I was very lucky and my year was not having Csaszar as lecturer rather the totally unknown young Laszlo Czach. After having both in elementary school second grade and in high school, two exceptional human beings as teachers, here in Uni again I was exposed to didactical honesty, to the belief that math can be understood by anybody.
I myself was still a moron and appreciated all this only instinctively. Only when I became a champion of anti Formalism did I realize these influences.

"Tide goes in, tide goes out" said the demagogue Bill O'Reilly a few weeks ago to the numb shocked David Silverman.
The sad part is that the well intentioned Silverman himself has no idea why the "tide goes in, tide goes out". He could have and then the whole conversation could have had a direction and meaning. Silverman probably defends his lack of understanding by the idea that "we can't know everything". But this is a just as demagogue defense as Reilly's "tide goes in tide goes out".
If you don't understand the tides, you have no idea about Newton Laws, mechanics, physics, math, nature and Man.
Life is only too short to learn about these things, if you are always too busy to do less important things.

Surface Acceleration Of Planets

If we ignore the different heights on the surface of a planet, then the weights are proportional to the masses and thus, the acceleration of falling bodies is constant.
We can calculate these accelerations by the law of gravitation from data of the planet.

Indeed, $a = \dfrac{F}{m} = \dfrac{\gamma \dfrac{mM}{r^2}}{m} = \dfrac{\gamma M}{r^2}$.

For the earth, $a = g = 9.8\ \text{m/s}^2 = \dfrac{\gamma M}{r^2}$.

The earth's radius is 6 400 km = 6 400 000 m, so from these we can calculate γM.

Surface Satellite, Or First Cosmic Speed

The required horizontal speed for Newton's cannon ball can be established by realizing that the a acceleration is also the circular acceleration with v speed and r radius. This we calculated as $\frac{v^2}{r}$.

Thus, the condition is $\frac{v^2}{r} = \frac{\gamma M}{r^2}$. From this, $v = \sqrt{\frac{\gamma M}{r}}$.

For the earth $\frac{\gamma M}{r^2} = g = 9.8 \approx 10$. And $r = 6400$ km $= 6\,400\,000$ m so,

$\frac{\gamma M}{r} \approx 10 \cdot 6\,400\,000 = 64\,000\,000$ and so $v = \sqrt{64\,000\,000} = 8\,000$ m/s.

This is also called as the first cosmic speed, because it is the minimal speed to launch a satellite around the earth.

Real Falling

The tides were derived from the realization that the differences in gravitation due to the different distances can not be ignored. We even exaggerated this in the vision of a falling water balloon, elongating its shape. Now we again try to use the exact gravitation, but quantitatively too. Namely, we want to calculate how a small body, that is point mass really falls if we regard its acceleration not to be fix, but rather increasing as it gets closer and closer to the ground.

Unlike at fix a acceleration, where the speed was the simple $v = v_0 \pm a\,t$, if the acceleration is changing, then the speed will be obtained, depending on not the time, but rather the height. This height of course, is actually a d distance from the center of the planet. The initial height will be a d_1 distance, and we'll use intermediate d_2, d_3, . . . distances to reach the final d_n distance which is actually the r radius of the planet. At every d_k distance we'll establish the new obtained speed, but in between, we'll cheat and regard the falling as it would happen with fix accelerations.

Of course, the accelerations at the different heights are already known from our Surface Acceleration formula above as $a = \frac{\gamma M}{r^2}$, but now $r = d_1, d_2, \ldots$

But we won't use these: $\frac{\gamma M}{d_1^{\,2}}, \frac{\gamma M}{d_2^{\,2}}, \frac{\gamma M}{d_3^{\,2}}, \ldots$ accelerations, rather an in between average value of them! Even more strangely, not their arithmetical average, rather their so called geometrical average. While the arithmetical average of two a, b numbers is simply $\frac{a+b}{2}$, their geometrical one is $\sqrt{a\,b}$. Thus, the first average will be $\sqrt{\frac{\gamma M}{d_1^{\,2}} \frac{\gamma M}{d_2^{\,2}}} = \frac{\gamma M}{d_1\,d_2}$. The next one $\sqrt{\frac{\gamma M}{d_2^{\,2}} \frac{\gamma M}{d_3^{\,2}}} = \frac{\gamma M}{d_2\,d_3}$, and so on.

The simple results show why the geometrical average was advantageous!

To calculate the end speeds for the cheated little intervals, we have to recap our formulas for motions with constantly changing speed:

$$a = \frac{v - v_0}{t} \rightarrow t = \frac{v - v_0}{a}$$

$$d = \frac{v + v_0}{2}\,t = \frac{v + v_0}{2}\,\frac{v - v_0}{a} = \frac{v^2 - v_0^{\,2}}{2\,a} \rightarrow v = \sqrt{v_0^{\,2} + 2ad}$$

Now we can start with the intervals!

From d_1 to d_2 we'll use the $\frac{\gamma M}{d_1 d_2}$ first average acceleration and of course, the $v_1 = v_0 = 0$ initial speed. Then, the end speed is

$$v_2 = \sqrt{0^2 + 2\frac{\gamma M}{d_1 d_2}(d_1 - d_2)} = \sqrt{2\gamma M\left(\frac{1}{d_2} - \frac{1}{d_1}\right)} .$$

This will be the initial speed in the next interval from d_2 to d_3, where we'll use the second $\frac{\gamma M}{d_2 d_3}$ average acceleration. Then, the end speed is

$$v_3 = \sqrt{{v_2}^2 + 2\frac{\gamma M}{d_2 d_3}(d_2 - d_3)} = \sqrt{2\gamma M\left(\frac{1}{d_2} - \frac{1}{d_1}\right) + 2\gamma M\left(\frac{1}{d_3} - \frac{1}{d_2}\right)} =$$

$$\sqrt{2\gamma M\left(\frac{1}{d_3} - \frac{1}{d_1}\right)}$$

And so on: $v_n = \sqrt{2\gamma M\left(\frac{1}{d_n} - \frac{1}{d_1}\right)} = \sqrt{2\gamma M\left(\frac{1}{r} - \frac{1}{d_1}\right)}$.

So this is the speed with which the body falls on the surface from d_1 initial height.
In reverse, this is the speed that is necessary, to kick up a ball up to d_1 height.

Speed To Infinity Or Second Cosmic Speed

What speed is necessary to kick up the ball to infinity? This may sound stupid, but we can change it to: With what speed is it necessary to shoot a capsule out of space so that it will never return? From our previous result:

$$v = \sqrt{2\gamma M\left(\frac{1}{r} - \frac{1}{\infty}\right)} = \sqrt{\frac{2\gamma M}{r}} .$$

For the earth $\frac{\gamma M}{r^2} = g = 9.8 \approx 10$. And $r = 6400 \text{ km} = 6\,400\,000 \text{ m}$ so,

$\frac{2\gamma M}{r} \approx 2 \cdot 10 \cdot 6\,400\,000 = 2 \cdot 64\,000\,000$ and so

$v = \sqrt{2 \cdot 64\,000\,000} = \sqrt{2} \cdot 8\,000$ m/s. This is also called the second cosmic speed.

Gravitational Paradoxes

Newton's solution of the ball problem was mostly motivated to calculate the earth's gravitation from its size and mass. We also followed this line and used it to calculate the gravitation, acceleration and exact fallings on planets. But the fact that the surface gravitation on an M ball with r radius is $F = \gamma \frac{m M}{r^2}$ can lead to quite amazing consequences.

We already mentioned that everyday objects around us all have gravitational forces, but these are so small, that they are undetectable. Thus, it is no wonder that we don't have a plausible feeling about gravitations at all. We know that the sun is massive and thus, it has a strong gravitational force. But still, the massive is associated with being heavy or dense. The δ density as we introduced it, not long ago, is the point by point measure of how the mass is changing. Namely, partitionally multiplying it with the volume gives the mass, $\delta \circ u = m$. Now if the density is the same at every point, then the partitional multiplication is simple multiplication, so $m = \delta u$. In other words, δ merely means the mass in a unit volume.

If we think of a very low density material, like a fluffy pack of cotton, then we associate this with also a small mass. But of course, if $m = \delta u$, then taking a large enough u volume, we can always achieve an arbitrary large mass. In other words, a big enough pack of cotton can be as heavy as we want it. To put it this way, sounds almost obvious and a bit silly. And yet, if we say that on the surface of a large enough cotton ball, the gravitation can be bigger than on the sun, that sounds quite unbelievable. But lets use the $F = \gamma \frac{m M}{r^2}$ formula for a cotton ball with r radius. If δ is the density of cotton then the cotton ball's M mass is δu, where u is the volume of the ball with r radius. We saw earlier that the surface of the sphere is increasing with the square of the radius. We showed this through little squares that can combine the surface. Similarly, the volume of a ball can be combined from the little pyramids that have these squares as base and the radius as height. Thus, of course, these little volumes will increase with the cube of the radius and so will the total of the ball too. In other words, $u = b\, r^3$ where b is a constant that we don't care about now. The main thing is that:

$F = \gamma \frac{m \delta b r^3}{r^2} = \gamma\, m\, \delta\, b\, r$. As we see, if r is increased sufficiently, the F gravitational force on a fix m mass will become arbitrary large on the surface of the cotton ball. This could be called the first gravitational paradox.

Strangely, when in the cosmos, gases combine into big clusters exactly this happens, what we just imagined for cotton. And yet, our assumptions were ridiculous about the huge surface gravitations. Indeed, before a cotton ball or cluster, could reach that size, the increasing surface gravitation pulls the ball itself together. But what does that mean? The M mass remains the same and only r decreases. So then, in

$F = \gamma \frac{m M}{r^2}$, only r decreases and so F increases even more. Thus, the strange becomes even stranger, the surface gravitation grows even bigger. This is the second gravitational paradox.

As the collapsing ball reaches a higher density, it becomes more rigid, but as we saw, the gravitation increases too, so it will be a race against each other. The strength of the matter versus the increasing gravitation. This is exactly how suns are born and solar systems and life come out of the simple gravitational force.

Yet instead of the details of how these gravitational collapses can proceed, we jump again to an extreme final situation, in the followings.

Black Hole

As we mentioned earlier, according to the Theory of Relativity, nothing can move faster than the speed of light. In fact, normal objects can never even reach this speed, while some special particles can only move with that and can never slow down. The particles of light, the photons, are one of these, just as the so called gravitons that carry the force of gravitation.

The cosmic speeds $\sqrt{\frac{\gamma M}{r}}$ and $\sqrt{\frac{2 \gamma M}{r}}$, were calculated for the earth, but we can just as well use these formulas to obtain them for any other planet or in general for any M-ball with r radius.

The previous gravitational paradoxes suggest that for large enough balls, something strange must happen with the cosmic speeds too. Indeed, if c is the speed of light, then from the above formulas, we can easily obtain M and r, so that the cosmic speeds become bigger than c. Lets remember, that these speeds were the required initial "kicks" for an object to become either a satellite or to leave forever. But, if nothing can move faster than c, then for such M, r balls, this must mean that nothing can orbit or leave that ball. Not even light! In other words, such balls wouldn't be even seeable. Thus, the name black hole is quite understandable.
Surprisingly, the idea of black holes came quite early. Reverend John Mitchell was the first to think of it and he even estimated that if the sun were five hundred times bigger, it would become a black hole.

M-Shrinking Radius To Black Hole, Mini Black Holes

The $\sqrt{\frac{2\gamma M}{r}} > c$ black hole condition allows different M and r combinations.
A more definite question would be that if we start with an M mass, then at what r would it become a black hole.

To get this, we merely have to express r from the above inequality. Squaring it and changing r and c, we at once get that: $r < \frac{2\gamma M}{c^2}$.

This means that any mass if compressed sufficiently, will become a black hole.
Using the earth M mass and the proper values of the γ and c constants, we would get about a centimeter for r. In other words, the whole earth's mass must be compressed into a centimeter ball to achieve a black hole.
This calculation doesn't mean that this compression can be achieved!
As we saw, from the gravitational paradoxes, a large enough mass, even if the density is low, can produce arbitrary large gravitation. So, it's quite expectable that with large enough M mass, a realistic black hole can appear. Maybe the earth's mass is too small to start with, to achieve a black hole. Still the idea that arbitrary small black hole could exist in theory, is an entertaining one. Some people say that if something is physically possible in theory, then sooner or later we'll find a reality of it too. I'm not so sure about this, but it's an interesting question in general.

δ-Blow Up Radius To Black Hole

Now lets calculate the more realistic situation, that we start with a fix δ density and ask how large ball with this density would become a black hole.

$$\sqrt{\frac{2\gamma M}{r}} = \sqrt{\frac{2\gamma\delta u}{r}} = \sqrt{\frac{2\gamma\delta b r^3}{r}} = r\sqrt{2\gamma\delta b} > c \rightarrow r > \frac{c}{\sqrt{2\gamma\delta b}}.$$

Lets remember that b was the proportionality of the ball's volume to r^3.

$$b = \frac{4\pi}{3} = \frac{4\cdot 3.14}{3} \approx 4, \qquad \text{so } r > \frac{c}{2\sqrt{2\gamma\delta}}.$$

If we use the proper values for the c, γ constants and the sun's density for δ, then this indeed gives very close to the radius that Reverend John Mitchell estimated.