Halting Calculus

The lambda calculus was invented before Turing came up with a system that directly captured all effective collections. In fact, this objective entity that I just called effective, was called earlier as a recursively enumerable set, relying on recursive functions.
The functional approach was the disguise behind which the recursive or more precisely, the dually recursive that is yes and no decidable collections were regarded as start.
As it turned out, gradually this approach recognized its own limitations.
The point by today is evident: Some decisions must be done by potentially non terminating steps!
Turing's particular word usage for the termination, that is stop was halt and this became accepted as homage to his ingenuity and I follow this too as the title shows.
Kleene was able to salvage the old functional approach by introducing the partial recursive functions. They should be called partial recursive partial function and I am not stuttering, merely tried to emphasize that they are not functions at all and this allows the more general recursion.
So actually the first partial should be replaced by general and indeed these were the general recursive partial functions. The point is of course that they are not functions.
The Recursion and Fix Point theorems were the first glimpses behind the amazing and necessary redundancies behind the effective collections.
Somehow, the Second World War became not just a Historical turning point but meant a mathematical pause too. It solidified some concepts too. Most importantly, Turing's other and more important word usage of computability. As is well known, he was involved not only in the German code breaking but further real computer research too. Then of course computers became the machines of the new age. So though the Turing Machine remained the accepted term, his system of effectivity lead to a very illogical renaming of recursive with computable.
There are many reasons that this push was an illogical, false and insane homage to Turing.
We must admit that Turing himself was a corporate in this.
His article title uses the expression "computable numbers" and we might think that he stresses the real originality of his own system and thus means natural numbers collected by his machines.
But unfortunately, NO! He meant real numbers and dual effectivity that is old recursiveness.
By the way, the second part of his title was also a huge mistake because it suggests that he solved the Entscheidung Problem which is not the case. That problem was inexact because there is no exact concept of Effectivity. There could be! Indeed, a general system that intuitively contains all effective methods. The intuitive then could be a subject of judgment but the reality is that none of the systems grab any even large portion of the effective methods. And that includes Turing's own machines too. So the claim that he captured effectivity, was not shown in his article at all.
This external universality was just a belief supported by the internal results.
Namely, a real and present internal universality! That there exist universal machines!
So his article title should have been: Universal Machines.
The elucidation is then that math is not dealing with matter and so these machines are actually tables of actions. The elucidation of action is then rewriting and reading position change.
This makes the word computer more justifiable for machine.
The real point is that his computers have an infinite working memory and all important results relate to this usage of the infinity. Thus these results have nothing to do with real computers.
In fact, the very essence of his article that there are universal computers seems trivial for real computers but it is a mirage. While for his theoretical computers it is not trivial at all, In fact, seems quite unbelievable! Unfortunately, students don't go through the messy details to create a concrete universal Turing Machine.
But this ugliness of seeing the power of Turing Machines is just one side.
The other side is actually a didactical paradox. Namely, that the important results of Kleene about effective collections did not become easier facts about Turing Machines.
In fact, a swindle is performed by using the partial recursive functions as visual road for the theorems but always claim that a Turing Machine representation can verify those steps.
So the avoidance of the old functional approach and natural numbers altogether is a must!

The beauty of texts as inputs that we alter letter by letter is fundamental.
Randomness could have become a reinforcer of this beauty but itself stepped back to numbers.
But the real importance of my new approach is to show that not only the forms of operations reflect the effective steps as it is seen in lambda calculus, but the theorems are also inherent in a correct formalism. And such correct formalism must revolve around the haltings.
So this is actually a first example of something that is still in a far future.
The existence of a Didactical Logic. When understanding becomes part of the subject itself.
This is my real motivation since that crucial day when I realized in high school that I am a mathematician. That happened after an elementary school period in which I was made to think about myself as a total failure. So getting into the math high school accidently and reluctantly obeying my mother's wish to even apply, could have been the reason for my other more general recognition: Everybody can understand everything that is understandable by humans.
In particular, the struggle to get inside math is not caused by anybody's inherent inability.
Realizing that the "I was bad in math" in truth means "I had a horrible math teacher" is the first step to start learning math.
And math is unavoidable! But not because it is behind the objective world, at all.
In fact, it is very hidden there! Yet evident in our intuitions!
So you can not be smart about anything if you can not move in math!
You can not understand Physics and can not make Philosophical claims either.
And again, not because these contain math! NO! It is the correct approach, to form meaningful questions and claims is that requires to be inside math. In fact, math is the most useless and yet most necessary human activity. But just as the army of bad math teachers turn children away from math, the army of bad mathematicians hide this very fact.
Wikipedia is the single and most evident player in degrading the sacred content of Understanding to a hollow collection of facts. The strive to be factual and not lie is very honest and rewardable.
But to be blind to the fact that deep down people want to understand is a fatal blindness.
The replacement of understanding is the symptom. With reading Wikipedia is obviously better than with reading conspiracy theories. But the point remains that dying without ever understanding anything correctly is eternal doom!

God doesn't care about what we think about him. There is only one single commandment!

Whatever God gave to you, you must give it back to others.

God doesn't want anything back to himself. No worship, not even belief in him!
The single payback to God is through enlightening others about stuff that you see.
Stuff that has nothing to do with God. In fact, talking about God is a sin.
The only exemption is this catch twenty two of exempting him from his only commandment.
So teaching is the only sacred activity, the road to salvation, defying the devil.
When someone gets something from God it is evident to the person yet for most it will not become an admission! And strangely, simply because the teaching doesn't start.
So this is a secondary catch twenty two, to keep the smartest people in the darkness.
The distinction of people as those who get transcendental help and those who don't is a taboo.
It is just one of the taboos that deny the existence of the soul.
Unlike God, about whom we can not talk descriptively, the soul is very concrete and describable!
Yet we use stupid synonyms for it like the heart. I just heard the religious health minister of Hungary say that while doctors are the brain of helping the patients, the nurses are the hearts and soul of the help. It matches his other claim that the Shroud Of Turin is an irrefutable proof for the transcendence of Jesus. Indeed, if someone needs a synonym for the soul, which is infinitely more concrete than the material heart, then no wonder he needs a fake object for his faith.
The descriptions of the soul involve talking about the consciousness and thinking.
The specific feature of thinking, that only humans are capable of, is another taboo.
Like all taboos it contains a catch twenty two again!
And indeed, the point is not that animals can not think rather that all humans can!

So lying that animals can think too, actually serves the devil by avoiding the universality of human thinking that leads to teaching and salvation.
Nowadays consciousness became a favorite topic. Dozens of half baked philosophical rubbish claims that solved the “problem”. But there is no problem in what makes consciousness!
The soul! So then the only fact that follows is that animals have soul.
Even religions accept this. The real puzzle is why animals can not think.
So thinking is an extra layer of consciousness. The moral consequences are of course fuzzy.
Can we eat something that has a soul? Religion took care of this with excuses.
But the more important moral consequence is to appreciate the second layer.
So to accept that thinking is an irrefutable proof that consciousness goes beyond matter.
Irrefutable because it is universal and repeatable, in fact happens always in any thinking.
So we humans live in a continuous state of miracle, that we abuse and ignore at the same time.
The abuse is the moral failing and the ignore is the intellectual.
Some people touched by God turn to religion which is a trap. But some, like I was in high school just remain in intellectual darkness. Accumulate the presents from God without questioning why.
That is where Philosophy can step in. Of course a proud arrogant young mathematician who even looked down on Physics is not a good soil for philosophy. So you need luck.
Like Hegel’s Lectures About The History Of Philosophy actually falling on your head.
The first line was enough to capture my attention and every following word was filling the void.
Gentlemen, we start the history of Philosophy, which is actually the history of Idealism because materialism is not philosophy just bad science! So then I became a Platonist.
This is more than becoming member of a sect because the sheer intellect leads one into.
Plato had no idea where consciousness might sit and indeed, his student Aristotle claimed the brain to be a blood cooling organ. But Plato was purer and stepped to the second level and thus avoided the mentioned problems about animals too. If you want to get out of matter, one door is enough and so you should choose the surest one. And for all humans it is thinking about thinking.
To realize that thinking can not be material is the simplest entry to Idealism, it is Platonism.
The Cogito Ergo Sum is just a syllogism attached to Platonism.
But those who believe that matter can produce thinking are neither stupid nor in sin.
Those who believe that there is no God, the atheists, are a more “named” class to ignore the real Rubicon of materialism contra Idealism. So this is again a catch twenty two, hiding taboos.

Back from these really important things to the machines, we must continue with what happened after the War. Amazingly, it happened already once before!
And this might help to see the importance of it.
The previous big idea was of course Cantor’s discovery of the sets.
Actually, only three fundamental conceptual discoveries happened since Newton.
Cantor’s sets, then Einstein’s realization that time is physical and finally Turing’s halting.
About the same time that Einstein made his recognitions, a second wave of understanding sets happened. It is called the Axiom Of Choice and the resulting Well Ordering Theorem.

These three people realized something that “was not in the air”. Yet they were not the kind of geniuses like Newton, Gauss and Gödel. Not saying at all, that these three super geniuses recognized only what was in the air. But I do claim that the Cantor, Einstein, Turing trio are counter players of the three super geniuses Newton, Gauss, Gödel.
They got something from the other side that these super geniuses could not!
Most importantly, while super geniuses can tell from the minutest facts the consequences, the deep geniuses can not even conceive how far their own discoveries will lead.
Einstein first rejected the four dimensional formalism. Very smartly, because it is beautiful not concretely just abstractly. Then he rejected the idea of the nuclear explosion, the gravitation waves and even the black holes. All consequences of his own theory. So the “second wavers” are crucial because they go deeper into the God given fundamental concepts! The Axiom Of Choice was the rebirth of Set Theory or we could even say that the birth of it after the naïve stage in the womb.
A very similar thing happened after the War but there was even a strange prelude to this.

The mentioned Kleene, who reformed the old functional approach also wrote a "Bible" that he very smartly titled "Introduction to Meta Mathematics". And this was just before what I call the real recognition of Turing's halting idea. Even more strangely, this new theorem was actually a simple consequence of the mentioned insightful Recursion and Fix Point theorems of Kleene.
And yet, the real insight was this new consequence! So here something different happened than back when Zermelo realized that we were blind to an already used axiom and so time can be avoided in Set Theory! Exactly when Einstein realized how to bring time into Physics.
Now in this crazier new age, this breakthrough remained as a mere easy consequence.
So the recognition of it as a paradigm change in effectivity was and is still not emphasized at all.
Not surprisingly, because then we must face that it was not in Kleene's "Bible" and not recognized by Gödel. And facing these, means that understanding is not about proving theorems.
So then of course math is also not about proving theorems.
Yet there can be no math without proving theorems.

The Wikipedia Science articles are an obscenity! Especially the math articles.
FACT: No math article is digestible by any high school math teacher!
The insane citations as requirements of acceptable academic scholarship is a menace.
Yes, I know it's an encyclopedia not a text book but without an understanding audience it is superficial. The handful of people that can follow it can gather their facts somewhere else too.
So it's time to name this new grand theorem that I compare to the Well Ordering Theorem.
It is Rice's Theorem and to prove my point about Wikipedia you can check it there.

There was an other parallel wave of second wavers that opened the lid on a deeper mystery.
Indeed, Rice's Theorem is more like a clarification. How to make sure that the opposite of something effective is not effective. But this other feature, the Effective Inseparability shows that such non dually effective pairs can have an internal cause already. It also goes back to the origins!
Indeed, the origin is what Turing didn't mention in his title neither in his few sentences as introduction. That he gave an alternative interpretation of the WHY. Why is Number Theory incomplete. Then of course it got a name, a pretty stupid name by the way as being undecidable.
So Gödel's language cause as being able to talk about itself got replaced by the more physical fact that the theorems are an effective set that is complicated enough to make the complement, the non theorems non effective. I remember when I realized this point from Rosser's little book.
But the Robinson Arithmetic shows a much deeper reality. Namely, that the theorems and the anti theorems of this weak system is a non separable set and so no extension can make it complete.
All these were far beyond what Turing saw and yet used not much more difficult steps.
Because it is the vision that makes everything possible. The understanding!
So the teaching must start with the obvious, the visible simple cause if that exist.
That's why the Well Ordering Theorem and it's parallel the Rice's Theorem are crucial.
The first is completely misunderstood as if it were entangled with the Axiom Of Choice.
Exactly the opposite is true! The Axiom Of Choice is only the final punch line.
Any function will expand by itself to a first non continuable stage.
This growth of the function is a very visual timely vision that can be defined spatially.
Apart from this, the Axiom Of Choice is also a spatial collection that goes beyond the old collection by properties. Namely, it is a random collection of a sample.
Zermelo not only discovered this hidden axiom but realized the previously mentioned "hard yard" in the old set theory without this new axiom.
Unfortunately, not using the concept of growth and thus mixing up his two achievements.
So this random spatial collection used as particular function that we allow to grow, gives the well ordering of a set. This vision is still not explained in any Set Theory book!
So why am I surprised that the newer corresponding situation is left in foggy formalism too.
I am not, and in fact know that the situation remains the same for a long time!
And yet, very reluctantly I will present the following system that goes up to the first level.
This means that we'll regard results only for machines that halt for sure and we'll call these transformers. They represent the old recursiveness or dual effectivity.

In this system machines and texts are the same!
You might think of programs but be careful because the formalism suggests a more surprising world where machines operate on machines.
More precisely, it will be action graphs altering action graphs!
But how such alteration should be defined is not known to me yet.
If you can figure out this then the formalism will have new meanings to you too.

IN TRUTH, THIS CHALLENGE IS THE REASON YOU SHOULD CONTINUE READING!

In his 1936 article, Turing clearly explains that he went back to Elementary School ideas.
And indeed, when kids do the digit by digit algorithms for the basic operations, then they use a squared paper and by writing single digits into them can obtain the correct results.
Now if we can obtain these operations by such primitive methods, then we can reverse this whole idea and ask what operations can be obtained by a generalized system of such digital calculations.
Can it be that by being general enough we can obtain everything that is effectively calculable?
The answer is yes and the amazing fact is that this generalness needs very simple possibilities.
Instead of the learnt rules how we add or multiply single digits and write them under each other, and so on, we can use a rectangular table of triplets that tells everything.
On one side, this is simpler but by allowing arbitrary big tables we still grasp more.
As start, we allow the ten digits 0 , 1 , . . , 9 to be m many 0 , 1 , 2 , . . . , m – 1 possible digit values but this is not the one that allows bigger tables.
Merely a convenience for later when we can replace these with arbitrary m many symbols.
These m many values or symbols will be written on our squared paper and will be also positioning the lines in the columns in our table.
For a concrete number or symbol underwriting system this m is fix.
So each column will have m many lines and the counting starts from 0.
The crucial potential increasing of our tables is the arbitrary n number of columns.
Here the counting is normal so the first column is 1 then 2 and so on.
So our tables are m n many triplets with m being fix but n arbitrary big.
The triplets themselves consist two numbers and a → or ← arrow between them.
Here is a concrete table:

1 → 2	1 ← 2	1 ← 3
1 → 0	0 → 3	1 ← 1

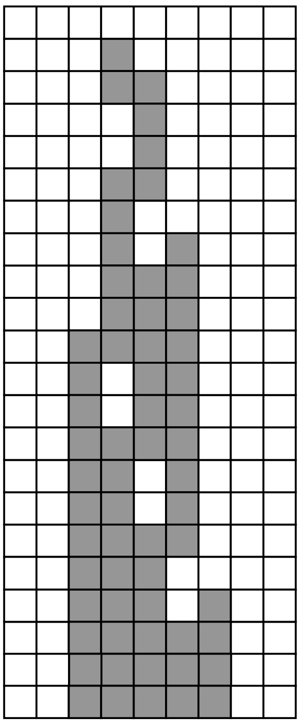
As we see, m = 2 and n = 3. The m = 2 means that we have only two symbols, or in simplest way two digits 0 and 1. This table will tell an action sequence carried out on our squared paper.
Namely, we will write one 0 or 1 digits in every new line under a fully filled starting line.
Now we regard the simplest scenario when our starting line is all 0-s.
So we can start from any square of this number line and use our table to write a new number in the next line. Quite logically, we use the first column of our table to start our actions.
Remember, we said that the m many number values will also denote the lines in every column.
So our table has the 0 , 1 number values as line numbers in our first column too.
Not surprisingly, they will correspond to the value in the square, under which we write.
Or put it an other way, this is the value we see above where we write.
Since we started from an all 0 line, we must use our 0 numbered, that is first line: 1 → 2.
The first of the triplet, 1 tells what to write in the new square.
The second, the → arrow tells that we should move one step in this direction and under this will be our new square to write in.
The third, the number 2 tells that we should now move in our table into the second column.
From here actually we repeat what we did previously.
We look up in our squared paper to see what the number value is above and go into this line in our table column. Since we started with all 0-s thus we see again a 0 above and so we go in our second column into the first line: 1 ← 2. So we again write a 1 in the new square.

But now we move left and so we will be under our previously written 1.
Of course, we must go a line down too and use the square under as next one.
The 2 tells that we must stay in our second column. To see what line to apply, we must look again up in our squared paper and as I said we'll see a 1 so we must use the second line:
0 → 3. Thus we write a 0 move to the right and under this will be our new square to write in.
The 3 tells that we should move into the third column and we'll use the second column since we see 1 above. That line is 1 ← 1 and so we write 1 again under the already seen 1 above.
We move left and move in our table to the first column.
The squared paper by now will look like this:

```
. . . . 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 . . . .
                           1
                            1
                           0
                            1
```

We might think that this would go on forever but observe that in our first column we have a 0 second number in the second row but there is no 0 column!
So this 0 means an end or stop or with Turing's original word usage, a halt.
But do we necessarily encounter this halt order?
NO! And this is the whole essence of Turing's ingenious system.
Some tables from some starting line do get to halt while others from other start don't.
In fact, this double dependence of the halt from both the table and the starting line is an other crucial feature. It reveals that some starting lines can replace the tables themselves as programs.

The under-writing action sequence is very clear because we see the whole history, what happened.
There is only one negative feature, namely that the blank squares made this possible.
We saw through these blank squares to see the last written values above.
If we want to use the empty blank content as symbol or value too and for example we chose 0 as this then we must update every new line with all the filled in 0-s or 1-s.
Then everything becomes messier, so we'll show now all the steps till halt, with white meaning 0 and grey meaning 1. Even with this, to see where we were at a step is a bit confusing.
A simple help is that if two consecutive lines differ at a place then the writing obviously happened there. Our last halting place will be not such and so to help a bit we mark it with an x:

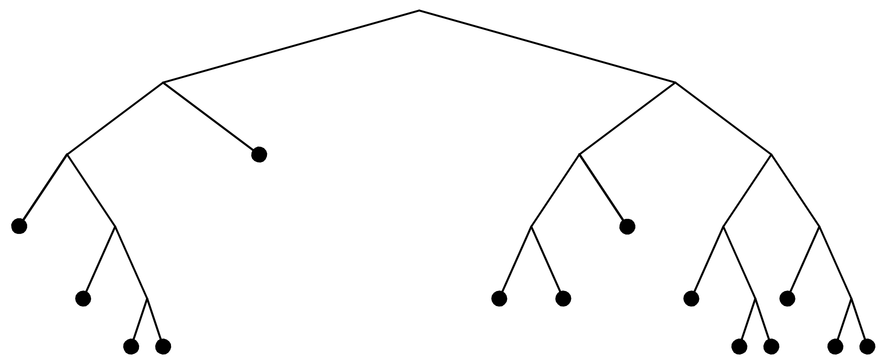


x

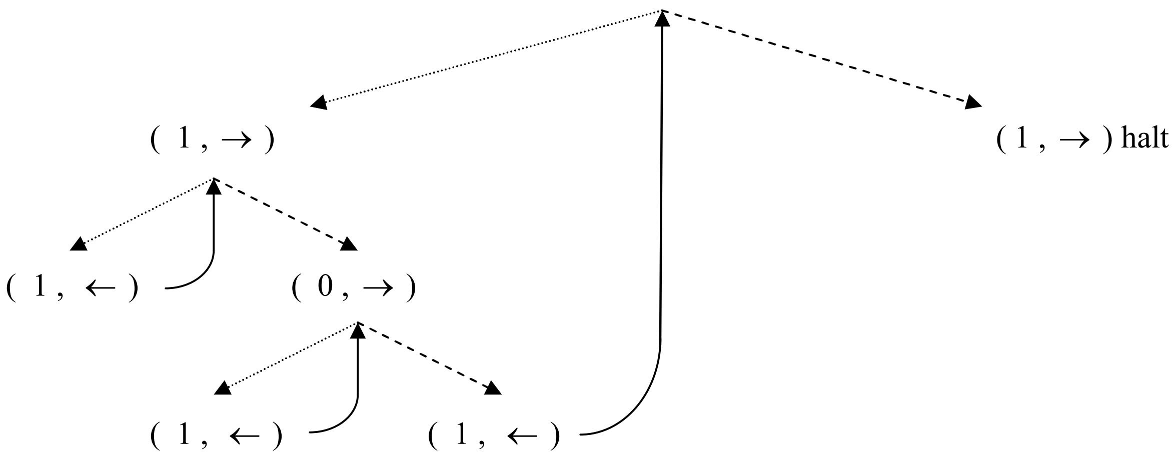
This updated blankless underwriting sequence was only pursued to come to the rewriting system that is the well accepted third step. Here we don't write the new symbols under the old rather we erase the old and write the new in its place. So we do not even need a squared paper just a line of squares. The elementary school solution could only be to use an eraser continually but the modern technical solution is to use a single line of memory cells!
This line being infinite means that actually we have an infinite memory and yet the single line means that we do not need memory registers and addresses.
The memory already suggests the further technical steps to turn a table into a machine.
We need a table head that does the jumps in the table and a memory line head that does the rewritings and single steps left or right. We also need a communication between them so the table head can get the last read symbol and the line head can get the new symbol and move direction.

The almost unbelievable fact is that this system can imitate all real computers even if we attach to them an infinite memory.

Now we'll show an "action tree" meaning for the Turing tables.
First here is a finite graph with end points marked with dots:



Such graph becomes a redirected graph if we redirect the dots, except one.
This suggests already that this will be the halt while the redirected ones the other columns.
But this is not quite so because the merely continuing branchings are also columns that simply go to the next column.
Most importantly though, the two branching lines correspond to the read 0 or 1.
And finally, we must place the memory line actions to the branchings to get an actual action tree.
Here is the action tree version of the above table with solid lines as read 0 and dashed as 1.



As you see, we marked no states at all and actually in this case they are unique anyway.
Indeed, the top is column 1 and this determines the next as 2 which again determines 3.

If we would want to introduce these as special flow charts then it is amazingly simple too. Remember that in general we have rectangles containing the actions and rhombuses containing the yes or no decisions. Here we make decisions prior any action but they are not yes or no decisions, rather for any possible number value. So we don't need rhombuses at all.
The rectangles are then the writings and movements, here in the parenthesis.

Flowcharts are crucial as the fourth Elementary School subject that is totally ignored.
I am talking about upper grade Elementary School because the first four year to learn reading, writing, counting and multiplying is done pretty well. Though I could mention some new problems with learning the times table. This is getting stretched to a longer and longer period.
I wrote all about these crazy things elsewhere in an article that I titled the Seventh Wonder.
This title refers to that prior to the just listed four abilities there are two learned before Elementary School, namely walking and speaking. So these six: walking speaking, reading, writing, counting and multiplying are abilities that thanks God are accepted as fix roads that everybody must be able to walk. We don't say on the street how well somebody walks. Roads are fix abilities that are not gradable. Actually, all understanding relies on such ROADS but we stopped at these six.
So that's why I stressed in that article the seventh as the solving of word problems.
This fix road is actually the initial road into Math. So no wonder that the devil wants to foggy this road and make the moronic education system to be blind to it.
When I finished Uni I needed money and started tutoring. That was when I turned to the standard text book of Larichev containing thousands of word problems. We never opened this book in the math high school because we entered math on much wider grounds. At this time when I turned to Larichev it simply opened my eye to the fact that with such well designed book how successful one can be. Seeing the importance of introducing variables and thus awakening a priori intuitions came much later when I bitterly accepted that this simple road exists yet somehow it still remains covered with the dust of ignorance. But even later when I returned to Hungary as a pensioner and saw that this Golden Book has been banished altogether then I had to write an article.
In truth, I was motivated by something else too! An old Russian educator in South America called Andre Toom took up the gloves and took the punches.
He lost, I lost and instead of fighting for the trivial, I spent time to walk on!
By the way, in Hungary the head of the Science Academy became a mathematician called Laszlo Lovasz who I knew very well from my high school years. A wonder kid of the few, from the same class in a famous math high school. I was attending an unknown that just started and that's why I was able to get in accidently as I mentioned before. But I won the competitions in sequence so my school was very grateful and I spent the most amazing four years there.
The Hungarian Television started a variety of Science Competitions with the finals broadcasted live in glass cages. Amazing stuff in a seemingly dark socialist era. I enrolled too but was not getting any reply to be selected to the finals. Lovasz won the math subject.
I cried a bit but got over. And then in first year at Uni when I was going for an Algebra Exam, my professor Ervin Fried said "you don't have to say anything and will get the high mark".
I hated algebra so was happy but looked more surprised. He showed a paper. My perfect test from years back for the TV competition. It got pushed back in his drawer.
I told this story just to ignore it because this is not why I hate Lovasz. I hated him back then for how he explained the solutions, how he held the chalk, how he talked, how he married his high school sweetheart, for what he was. Now I hate him for selling out the Academy to a Dictator and most of all, accepting that the Academy has nothing to do with reforming the math education.
But most of all, accepting that the book Larichev is out of print for decades! An OBSCENITY!
But back to the positive, I was able to clearly pinpoint the eighth, ninth, and tenths miracles too.
And this brings us back to the "flow charts".
The eighth miracle is the "Grid System", introducing vectors, the ninth is " Grammatics" introducing the quantors and the tenth is the "flow charts" introducing Effectivity.
All these are must ROADS! Simple clear visual avenues to awake the common mathematician in everybody. Of course, they have to be denied even of their existence.

Halting Calculus, A texts only approach to Effectivity.

P , Q , R , S , T are variables for text, program, machine, effective property.
Θ is a variable for special machines, called transformers.
U is a fix constant machine.
Δ , Φ , Γ , Π , Σ are fix constant transformers.

$S\downarrow P$ = S halts in P
$S\uparrow P$ = $\neg(S\downarrow P)$ = S doesn't halt = S runs in P (forever)
[P] = { S ; $S\downarrow P$ } = effective set
$\neg$[P] = complement set of [P] = all the texts that are not in [P].
$\neg$P = complement machine of P is such that [$\neg$P] = $\neg$[P].

For most P this complement doesn't exist!
That is, the [P] effective set's complement is usually not effective.
All the followings are a clarification of this claim.

1. There is a | text coding that combines two texts into S|T.

The simplest solution to separate two texts would be to regard our | itself as a special symbol. Unfortunately, by this we increase our alphabet and so we couldn't claim results for any fix alphabet. Luckily, there is no need for this and a simple idea similar to what Turing used in his original idea is to double the symbols of S and then use a pair that has two different symbols.
Even the simplest dual 0 , 1 alphabet is enough for this where thus the separator is 01 or 10.
The machine that wants to use S|T can then start the text from left and regard only the single of the doubled symbols until a first non doubled as S and then read the next T section as it is.

2. There is a U universal machine that imitates every P because: $S|P\downarrow U \leftrightarrow S\downarrow P$.
We introduce the Θ variable that represents machines that halt from all inputs.
The Θ refers to transformer because they are actually effective text transformers.
Since for these the halting is sure we omit $\downarrow$.
Also, we can continue the transformer with an equal sign and tell the result: $S\Theta = R$.
Usually, we will not use such equal signs because we'll use these results further.
Then the transformer's continuation means using the result of it automatically.
Such continuation can be claiming a halt or run : $S\Theta\downarrow P$ or $S\Theta\uparrow P$.
But can be also a new transformer : $S\Theta\,\Theta'\downarrow P$.
In a sense they are like multiplication while the | coding was more like an addition.
This reflects our agreement about their hierarchy level that transformation overrides |.
So $S|T\Theta\downarrow P$ automatically means $S|(T\Theta)\downarrow P$ while for $(S|T)\Theta\downarrow P$ we can not omit the parenthesis.
Observe that our transformer continuation actually hides an assumption:

3. Transformer imbedding. $S\Theta\downarrow P \leftrightarrow S\downarrow \Theta P$

Thus ΘP can become a new text and then it can be also followed by a transformer so we need parenthesis: $(\Theta P)\,\Theta'$.

4. There is a Δ transformer that can create a coded double: $T|T = T\Delta$, so it is a duplicator.
Of course, the actual claim is about haltings for a P and then it can be continued with using 3.:
$T|T\downarrow P \leftrightarrow T\Delta\downarrow P \leftrightarrow T\downarrow \Delta P$.

Applying this in reverse with P as U :
$T\downarrow \Delta U \quad \leftrightarrow \quad T\Delta \downarrow U \quad \leftrightarrow \quad T|T\downarrow U \quad \leftrightarrow \quad T\downarrow T.$

Doesn't seem interesting but observe the followings:
We can call an T input as "shared" by two P , Q machines if they both halt or not from T.
Now if a Q is $\neg P$ then and only then P and Q can not share any T input.
So if a P shares input with every machine, then it can not have a complementing machine!
By our result, ΔU shares an input with every T machine, namely T itself and so ΔU is a machine that has no complement.
Of course, we can establish a non existing complement by deriving a contradiction with already proved cases too. For example, we can show that U itself can not have a complement.
Indeed, suppose it had the $\neg U$ complementing machine. Then we had:
$T\downarrow \Delta U \quad \leftrightarrow \quad T\Delta \downarrow U \quad \leftrightarrow \quad T\Delta \uparrow \neg U \quad \leftrightarrow \quad T\uparrow \Delta \neg U.$
And so ΔU had the $\Delta \neg U$ complement that we proved not to exist.
Non indirect arguments exist too but for those we must guess the shared input for arbitrary T.
In our case it is $\Delta T\,|\Delta T = (\Delta T\,)\Delta$ because:
$\Delta T\,|\Delta T\downarrow U \quad \leftrightarrow \quad \Delta T\downarrow \Delta T \quad \leftrightarrow \quad (\Delta T)\Delta \downarrow T \quad \leftrightarrow \quad \Delta T\,|\Delta T\downarrow T$
Or:
$(\Delta T)\Delta \downarrow U \quad \leftrightarrow \quad \Delta T\,|\Delta T\downarrow U \quad \leftrightarrow \quad \Delta T\downarrow \Delta T \quad \leftrightarrow \quad (\Delta T)\Delta \downarrow T$

Soon we'll see an infinity of shared inputs with U without guessing.

5. Partial input alteration.
The machine continuation for a combined S|T input means that $(S|T)\Theta \downarrow P \quad \leftrightarrow \quad S|T\downarrow \Theta P.$
If only one, say the second part of the input is altered by an Θ transformer then the building of Θ into a next used P machine is more complicated but still obtainable by a $(\Theta\,|P)\Phi$ machine.
So: $\qquad S|T\Theta \downarrow P \quad \leftrightarrow \quad S|T\downarrow (\Theta\,|P\,)\Phi\,.$
A particular example of Θ could use a doubled T|T but we could still build Θ into P.
So: $\qquad S|(\,T|T\,)\Theta \downarrow P \quad \leftrightarrow \quad S|T\downarrow (\,\Theta\,|P\,)\Gamma\,.$

These Φ , Γ are concrete fix machines. The more important Π comes next and finally Σ.

6. Parameter Theorem.
An other reason for regarding the input as combined S|T can be not that we alter T rather we want to get rid of it completely! Meaning that we want to build it into a next used P.
So: $\qquad S|T\downarrow P \quad \leftrightarrow \quad S\downarrow (\,T|P\,)\Pi$
So this fix Π imbeds T as a parameter in the new program that will use only S.

Recursion Theorem.
For every P there is a P^U that for all S : $\quad S|\,P^U \downarrow P \quad \leftrightarrow \quad S\downarrow P^U.$

The notation makes sense since P^U is like a "personal" universal machine for P.

A concrete such P^U is $(\,(\Pi\,|P)\Gamma\,|(\Pi\,|P)\Gamma\,)\Pi$. Indeed:

$$S|(\,(\Pi\,|P)\Gamma\,|(\Pi\,|P)\Gamma\,)\Pi \downarrow P \quad \leftrightarrow \quad S|(\Pi\,|P)\Gamma \downarrow (\Pi\,|P)\Gamma \quad \leftrightarrow \quad S\downarrow (\,(\Pi\,|P)\Gamma\,|(\Pi\,|P)\Gamma\,)\Pi$$

The first $\leftrightarrow$ is true by Γ's definition in 5. with $T = (\Pi\,|P)\Gamma$, $\Theta = \Pi$.
The second $\leftrightarrow$ is true by the Parameter Theorem with T and P both being $(\Pi\,|P)\Gamma$.

As we said, P^U is like a personal universal machine for P. But continuing our result to involve the real universal machine U : $\quad S|\,P^U \downarrow P \quad \leftrightarrow \quad S\downarrow P^U \quad \leftrightarrow \quad S|\,P^U \downarrow U.$

And so any P shares all the $S|P^U$ texts with U.
Unfortunately, it still just proves that U has no complement. So now we'll step beyond this.

Fix Point Theorem.
For every Θ transformer there is a Q program that: $[Q] = [Q\Theta]$.
That is: For every T input $T{\downarrow}Q \leftrightarrow T{\downarrow}Q\Theta$.

Let P be such that for all S , T we have $T{\downarrow}S\Theta \leftrightarrow S|T{\downarrow}P$.
Then let P^U be the text guaranteed by the Recursion Theorem. Then:
$T{\downarrow}P^U \leftrightarrow P^U|T{\downarrow}P \leftrightarrow T{\downarrow}P^U\Theta$. So such Q is P^U.

The first $\leftrightarrow$ is true by the Recursion Theorem.
The second $\leftrightarrow$ is true by our definition of P with $S = P^U$.

Two P , Q texts are variants if they collect the same texts: [P] = [Q].
A **T** set of texts is variant complete if with any member, the variants are members too.
If a set is variant complete then its complement is variant complete too, so actually any separation of all texts can be variant complete.
Rice's Theorem:
Let a variant complete separation of all texts be **A** , **B** with neither being empty.
Then they can not be both effective.

Let $A \in \mathbf{A}$, $B \in \mathbf{B}$ and define an Θ text function as: $T \in \mathbf{A} \to T\Theta = B$, $T \in \mathbf{B} \to T\Theta = A$.
If both **A** , **B** were effective then this Θ were a text transformer and then by the Fix Point Theorem for some Q text we would have that $[Q\Theta] = [Q]$.
Thus $Q\Theta$ and Q are variants and so they should be in the same of either **A** or **B**.
But by the definition of F we have that $F \in \mathbf{A} \to Q\Theta = B \in \mathbf{B}$ and $Q \in \mathbf{B} \to Q\Theta = A \in \mathbf{A}$.

Our second proof will tell exactly a side of the two variant complete separation which is non effective for sure. Namely, the side that contains all never halting programs.
We show that if this side were effective then all programs had a complement.
So suppose N would halt from all never halting programs and a member of the other side were P.
Then:
For all S $[S{\downarrow}P \leftrightarrow S{\downarrow}R] \to R{\uparrow}N$ and
For all S $[S{\uparrow}R] \to R{\downarrow}N$.

Now we claim a P&Q machine that uses S|T combined input and halts as:
$S|T{\downarrow}P\&Q \leftrightarrow S{\downarrow}P$ and $T{\downarrow}Q$.
To achieve such machine we simply have to run one of them and if that halts start the other.
The better usable two facts for our proof are:
$T{\downarrow}Q \to$ For all S $[S{\downarrow}P \leftrightarrow S|T{\downarrow}P\&Q]$ and
$T{\uparrow}Q \to$ For all S $[S|T{\uparrow}P\&Q]$.
Using for these both the Parameter Theorem, then the two N conditions and then again the Parameter Theorem:
$T{\downarrow}Q \to$ For all S $[S{\downarrow}P \leftrightarrow S|T{\downarrow}P\&Q \leftrightarrow S{\downarrow}(T|P\&Q)\Pi] \to$
$(T|P\&Q)\Pi{\uparrow}N \leftrightarrow (T|P\&Q){\uparrow}\Pi N \leftrightarrow T{\uparrow}(P\&Q|\Pi E)N$.

$T{\uparrow}Q \to$ For all S $[S|T{\uparrow}P\&Q \leftrightarrow S{\uparrow}(T|P\&Q)\Pi] \to$
$(T|P\&Q)\Pi{\downarrow}N \leftrightarrow (T|P\&Q){\downarrow}\Pi N \leftrightarrow T{\downarrow}(P\&Q|\Pi N)\Pi$.

So the complement of Q would be $(P\&Q|\Pi N)\Pi$.

The main application of Rice's Theorem is if we know one side that is effective for sure.
To get such side can be helped by our next axiom:

7. There is a Σ transformer that $S{\downarrow}P \leftrightarrow P{\downarrow}S\Sigma$.

We could claim a Π' analogue of Π that uses the first variable in S|T as parameter.
That is: $(S|T){\downarrow}P \leftrightarrow T{\downarrow}(S|P)\Pi'$. Then: $S{\downarrow}P \leftrightarrow S|P{\downarrow}U \leftrightarrow P{\downarrow}(S|U)\Pi' \leftrightarrow P{\downarrow}S\Sigma$.
So Σ is doing Π' but with the U universal machine built in fix.

For any S the [$S\Sigma$] collection is variant complete! That is, for every P , Q we have that:

($P{\downarrow}S\Sigma$ and [P] = [Q]) $\rightarrow$ $Q{\downarrow}S\Sigma$. Indeed:
By 7. $P{\downarrow}S\Sigma$ can be replaced by $S{\downarrow}P$.
[P] = [Q] means "for all T ($T{\downarrow}P \leftrightarrow T{\downarrow}Q$)".
Thus the ( ) condition trivially implies $S{\downarrow}Q$.
Finally, by 7. again we get the conclusion from this.
So by Rice's Theorem $S\Sigma$ has no complement.
In fact, $S\Sigma$ shares some P input with every Q and this has an amazing meaning.
First of all, the exact meaning is that:
For all S , Q there is a P that $P{\downarrow}S\Sigma$ and $P{\downarrow}Q$ or $P{\uparrow}S\Sigma$ and $P{\uparrow}Q$.
That is, $S{\downarrow}P$ and $P{\downarrow}Q$ or $S{\uparrow}P$ and $P{\uparrow}Q$.
So then P is actually a counter example for Q if it were intended as a machine recognizer of P not collecting the S member. Indeed, then for all P we should have that:
$S{\uparrow}P \leftrightarrow P{\downarrow}Q$ which is the exact opposite of what we obtained.
So even such simple behavior that a fix S doesn't halt in P, can not be perfectly recognized by an other machine examining P. The " perfectly" is the essence of course!
Because we could make a Q that halts if S doesn't halt in P by for example making Q halt from everything! But then the reverse is not true that Q only halts for these P.