Light and Time, Basic Relativity

Introduction

Einstein as a child was wondering about the following strange idea:
If we could leave the Earth with a rocket going faster than light then looking back we could see the past. More exactly, the images of events that happened before the launch of our rocket.
As an adult he realized that this dream is impossible. No object can go faster than light.
But the truth is stranger than the dream was. In a sense, no object can even chase the light.
An object can go with any speed under the speed of light and thus from the source of the light signal would seem as chasing the light signal but the light will still move with the same c speed relative to the chaser just as it moves relative to the source. This somehow feels contradictory to the simple fact that the chaser moves between the source and the light signal forever.
This geometrical in-betweenness is indeed a fact for both observers, the source and the chaser.
What we can feel at once is that such fixation of the c speed must mean that actually the time intervals or distances or maybe both must alter by different observers.
The truth is "both" plus an additional third fact that makes this whole observational relativity the strangest. Namely, that for any two observers the distortions in each other's observational values are symmetrical too. So each can think of the other as measuring similarly distorted values.
Thus the word "relative" in Relativity is meant in this actually opposite "absolute" sense.
In fact, this illogical naming goes back to an even wider absoluteness as relativity.
Namely, that an observer moving with a fix speed can not tell from experiments inside its moving laboratory his speed. Galileo was the first who dropped objects not only from the Pisa tower but inside a moving boat and realized that the boat motion does not affect the measurable facts.
It is strange that later Newton's mechanics gave an explanation for this absoluteness.
Indeed, in a fast moving train we can pour champagne in our glass because the liquid has the forward speed of the train too. So the seemingly same process has an explanation.
Of course, we can argue that actually even on the train platform the champagne shouldn't be falling into the cup "naturally" since the whole Earth is in motion. So an absoluteness of translational fix speed laboratories is something more basic than even Newtonian mechanics.
But this whole approach has serious problems. What is an inside experiment? Because looking out from a lab and observing the universe we obviously can see differences between the labs. One sees the Earth, the other is far away in deep space. And if we try to avoid this as trivial closing out of the outer world, we get into the deepest contradiction that actually started Relativity.
Namely, that letting light enter our lab but not regarding it as information can be allowed!
That extension of Galileo's relativity principle was accepted as the new Einstein version.
Behind this lies the fact that light is not like champagne that picks up a speed but neither is like sound that is carried in the air. Most amazingly, this absoluteness of light is with us quite trivially!
Indeed, imagine if light would be altered by the motion of the source or would be carried by a medium behind all matter. We would have to experience all kinds of "light shows" in both case!
But we never see anything "strange", which is of course the strangest if we think about it!
Einstein was the first to think about it and thus to realize this big miracle of light.
Even a decade earlier the Michelson Morley experiment tried to bounce back and forth light rays to see some difference as we turn the setup. But nothing happened! You can not establish your motion in the universe by using light! There is no medium in the universe that carries light so it travels like little bullets and though it has little hits and obeys gravity too accordingly, it can not be slowed down or sped up as bullets can. So the mechanics and geometrical motion of light is something new. It actually necessitates a geometrization of all mechanics.
And Einstein indeed achieved later a geometrization of the most important force, gravity.

Prelude, Roemer

The speed of light was established by Roemer through a magnificent idea based on the realization of Galileo that Jupiter has moons. The best visible one from Earth is called Io and has 42 hours orbiting time which is pretty fast compared to our moon's 28 days orbiting time.
This 42 hours is like fix far away clocking time that we obviously see with a delay due to the light taking some time to reach us. This still doesn't reveal the ingenious idea only that if we had someone right there at Jupiter and could communicate instantaneously with him then knowing the distance of Jupiter from Earth we could establish the speed of light quite trivially.
First of all such instantaneous communication is impossible. The speed of light is a limit of all information travellings. The other part is easy! We don't only know Jupiter's distance but we know how this distance is changing. And that is already a hint toward the big idea.
Jupiter is much further from the Sun as we are and so it orbits also much slower. In other words, its "year" is much longer than our actual year. So under the half year when we move from a closest distance to the farthest, Jupiter itself will move not much and so the distance difference is about the diameter of Earth's orbit which is 300 million kilometer. If you know the speed of light being 300 thousand kilometer per second then something strange should be obvious at once.
This diameter is thousand times the speed of light, so light should travel it in thousand seconds.
Roemer was able to measure this thousand seconds but could not quite convincingly explain his result and so many, including his boss at the Paris observatory didn't accept it.
But many did, including Newton who obviously pondered on Roemer's idea a lot but couldn't explained it in a better way either. Strangely though he arrived by some other arguments too at the result that the light from the Sun would travel in about eight minutes to Earth. Almost perfect match with the thousand seconds for the diameter measured by Roemer.
So how did Roemer measure this thousand seconds?
The big idea is that we synchronize a clock of our own to Io's period when we are closest to it and after six months we look at Jupiter again and will see a delaying of the period with thousand seconds. This thousand seconds is a measurable time relative in a 42 hour period with any fairly built clock but observe that the 42 hour interval also had to be kept repeating for the half year.
The clocks built at that time were good enough for this too. But the most important point is that this six months as time is irrelevant in the meaning of the delay. The delay purely comes from the backwards motion from the synchronized position to a farther one. Indeed, we get farther from the original source too with 1000 seconds extra delay. How slowly and on what path we move backward is irrelevant. However a second problem emerges about the backwards motion's length.
Namely, what if go back so much that the gradually building delay exceeds the whole 42 hour period. This is a fundamental problem in all synchronizations. If we synchronize our watches and promise to meet at nine for a dinner but you get drunk and fall asleep then the same happens.
You might come the next day believing that you will be on time. In Roemer's case this was impossible since the light in 42 hours would have travelled far out of the whole solar system.
But how could he know this then? And that's why he was not convincing.
Well, the ancients already knew that hundreds of kilometers per second is impossible for the light since they used mirrors between mountain peaks and saw no delay at all. If the actual speed of light were thousands or ten thousands of kilometer per second then indeed a 42 hour extra delay could have happened and indeed the unbelievers said that Roemer's result was too fast.
Gradually of course it turned out that Roemer was right. There was no overflowing of the delay.
By the way, with continuously watching Jupiter for the half year backtracking this could have been established too.

Perpendicular independence, light time dilation formula

Amazingly and even more amazingly never mentioned, the start of relativity is an already earlier never emphasized fundamental fact behind forces.
Newton in the second edition of his "Principia" placed a picture of the now classic "Cannon on Mount Everest". Possible projectiles show how faster and faster launch speeds will eventually make the ball return to the cannon and thus turn it into an artificial satellite of the Earth.
So orbiting is merely a falling with big enough side speed. Nice but something is missing!
We shoot a cannon on a straight field perfectly horizontally. The ball will move forward with its launch speed and would go forever if we neglect two things. The air resistance and gravity.
This second is the important now because it only effects the vertical motion. In other words, the ball is simply falling down as if we would have dropped it but at the same time keeps going forward. The result is a parabolic path, namely a very narrow segment of a parabola.
The Earth is of course not flat, so the parabola becomes a circle.
Namely, through using vectors and regarding limits. The details of these are irrelevant now.
In the introduction I revealed that both time intervals and distances must be distorted to get a fix speed of light. To get only the distortion of one of them we must apply some tricks.
The simplest is Einstein's original approach and the trick is the just described perpendicular independence. A distance perpendicular to the motion is not distorted.
So a horizontal speed does not affect the lengths of the vertical distances.
But such vertical distance can be used to measure time. Namely, by initiating a light signal at the bottom and returning it by a mirror at the top.

The time of the return trip along this d distance is $\frac{2d}{c}$.

If this "light rod" device is moving horizontally to the right with a v speed relative to an observer at rest then he will see the return of the signal not at the same place as it was released.
More importantly, the light trip will not be an up and down path rather slanted up and down.
In other words, as the two s sides of an equal sided triangle.

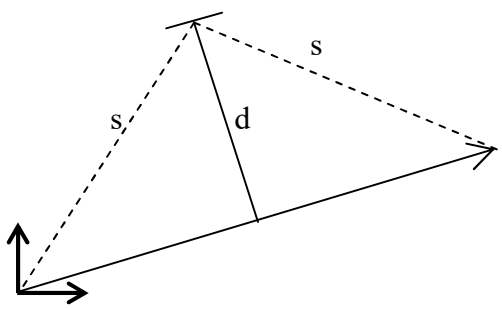
Light having a fix speed then means that for this observer the return time is $\frac{2s}{c}$ and so the base of the triangle seems $\frac{2s}{c}v$ long. By the perpendicular independence the height remains d and so half of the triangle is a right angle triangle with d, $\frac{sv}{c}$ sides at the right angle and s across.

So by Pythagoras Theorem: $s^2 - \left(\frac{sv}{c}\right)^2 = d^2$.

Dividing with d^2 we get: $\left(\frac{s}{d}\right)^2 - \left(\frac{sv}{dc}\right)^2 = \left(\frac{s}{d}\right)^2 \left(1 - \left(\frac{v}{c}\right)^2\right) = 1$ and so:

$\frac{s}{d} = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma$ Observe that this $\frac{s}{d}$ is also $\frac{\frac{2s}{c}}{\frac{2d}{c}}$ the ratio of the light times.

So the moving light time dilated with the γ factor for an observer at rest.

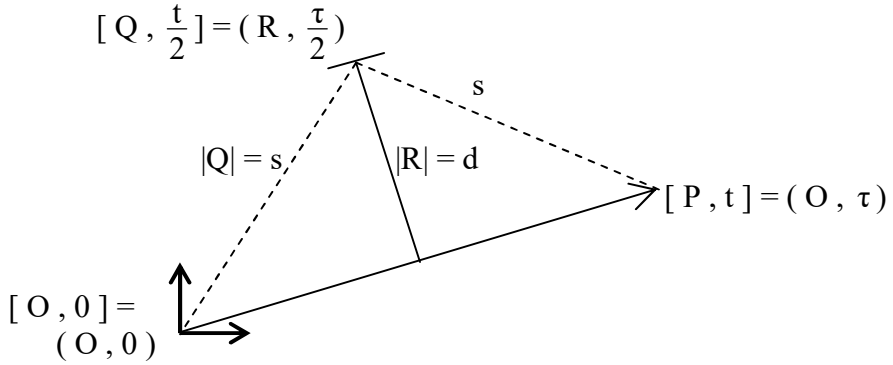


Self time formula, transformations

The relative v speed was taken as given above in the light time dilation formula.
By our remarks about the symmetry of relativity we can imagine that the exact same argument can be repeated in reverse and so a light time for a moving observer is also dilated by γ .
In fact, even their relative v speed should be the same though if the two systems are having coordinate systems in which the other is moving then of course the relative speeds are V and –V.
This is an important condition in getting the relativistic transformations but is not the main one.
For that we must somehow involve the distances that we intentionally avoided to get the γ time dilation factor. So now a new second heuristic idea will actually involve not just distances but the full P location of an event in a [P,t] system that we want to transform into an other as < P', t'>.
For simplicity we imagine all systems having a common initial position of their origins which is also the zero time in all of them. But this leaves a missing crucial point. Namely, how the times at other P locations should be synchronized to the O origin. Again Einstein's original idea is the simplest. We send light signals from the halving point to O and P.
So the combined argument is this:
We send the synchronizing signal from the middle point to O and P that starts the clocks there.
At the starting of O's clock we initiate a light rod to move towards P with a fix V speed, and being perpendicular to V's direction and initiate its light signal too.
We'll assume that this light stick's d length and its V speed are chosen so that the light signal's return to the bottom of the rod will happen exactly at P.
So in the [,] resting coordinate system the light stick reaches P exactly as the returning light reaches P too. The travelling stick of course feels as the light would go along it and its clock at the bottom will show a τ time. By the time dilation formula this is a longer $\gamma\,\tau$ time in [,].
If our synchronization process was correct then this should be exactly t. And thus:

$$\tau = \frac{t}{\gamma} = t\sqrt{1-\left(\frac{|V|}{c}\right)^2} = t\sqrt{1-\left(\frac{\frac{|P|}{t}}{c}\right)^2} = \sqrt{t^2-\left(\frac{|P|}{c}\right)^2}\,.$$

In our next figure the light rod is depicted as a (,) system:



Now comes the big idea that this self time can not depend on a system that we view it in and so:

If [P, t] = < P', t'> then also: $\sqrt{t^2-\left(\frac{|P|}{c}\right)^2} = \sqrt{t'^2-\left(\frac{|P'|}{c}\right)^2}$. Which leads to that:

$P^2-(ct)^2 = P'^2-(ct')^2$.

So $x^2+y^2+z^2-(ct)^2$ is an invariant value for all coordination of an event.

This invariance together with the invariance of the light speeds and the mentioned equal relative speeds can give the [P, t] $\rightarrow$ < P', t'> transformation if we also assume that it is linear.
Meaning that: x' = a x + b y + c z + d , y' = e x + f y + g z + h t and so on, including for time.

So we can claim exactly sixteen equations for these sixteen unknown a , b , . . . coefficients.
There could be many observations about the obtained transformations.
The first should be that they are called Lorentz transformations and were known before Einstein.
Poincare the mathematician pondered a lot about these too and even reached the famous $E = mc^2$ consequence. To me the most important episode is a much later one, long after Einstein published his new approach as Special Relativity. Someone told Lorentz that actually he should be the father of this theory since he realized the transformations. He said that this is totally incorrect because he only regarded the transformations of distances as possibly real while Einstein discovered that time is not absolute and this new vision would have led him to the concrete transformations anyway.
Beyond Lorentz's impeccable character this reveals more important points:
Firstly, that at this time Lorentz still didn't really grasp what his own transformations meant!
After the Michelson Morley experiment to contemplate that maybe the brass equipment is contracting under its turning and thus makes the apparent speed of light fix, was understandable.
But by the time Lorentz made his remark it was clear that the whole concept of real-apparent was attacked by Special Relativity. Strangely, then General Relativity seemed to bring in a new "real".
So degraded this side of the Special Theory. Not really but this is an even deeper story.
The sad fact is that this most important aspect of the Special Theory is still not understood by most of those who try to explain it.
The second most important fact about the transformations is actually the root of this first.
It is the simple observation that Einstein's second trick, the synchronized starting of the clocks in $[\ ,\ ]$ at O and P was the most important yet hollow step. Indeed, we never used the middle fact.
Only our belief that the t time at P is reliable as arrival time of the light.
My third observation is again general, actually about what we didn't mention yet. The paradoxes.
Relativity is paradoxical from the very start and those who draw red lines from where they decide to call things paradoxical are delusional.
But those who claim resolutions of these paradoxes are even more delusional with bigger egos.
Anybody who regards a travelling light signal as a travelling object and accepts the fix speed of these particular travellings, already has a fundamental paradox in his vision.
To learn the facts and play the game of reasoning by them then can lead to a game of pretend too.

But now we must see what the claimed derivable transformations look like in the simplest case.
This is when $y' = y$, $z' = z$ so the two systems are travelling with a relative v speed along x.

$$[\ x\ ,\ t\ ] \qquad \rightarrow \qquad < \gamma\ (\ x - vt\ )\ ,\ \gamma\ (\ t - \frac{v\,x}{c^2}\ ) >$$

Checking how it gives back the time dilation formula:

$$[\ v\ ,\ 1\ ] \qquad \rightarrow \qquad < \gamma\ (\ v - v\ 1\ )\ ,\ \gamma\ (\ t - \frac{v^2}{c^2}\ ) > \ = \ < 0\ ,\ \gamma >$$

Now with a $w > v$ speed we get:

$$[\ w\ ,\ 1\ ] \qquad \rightarrow \qquad < \gamma\ (\ w - v\ 1\ )\ ,\ \gamma\ (\ t - \frac{v\,w}{c^2}\ ) >$$

Dividing the x coordinate with the time coordinate we get $\dfrac{w - v}{1 - \dfrac{vw}{c^2}}$ which is the w_v speed that to a v speed object a faster w speed object seems to go with.

This is the "chasing" speed we mentioned earlier and now it turned out to be bigger than $w - v$.
Strange, but observe that necessarily because it still gives for $w = c$ the necessary c.

Indeed: $\dfrac{c - v}{1 - \dfrac{vc}{c^2}} = c$

Bigger pictures

You might think that for this we go into General Relativity. But that would only help if we wanted to include gravitation. If you know how Einstein involved gravitation by "equivalence" then you might defy this remark. Because equivalence means that an accelerating system can also be viewed as being in a gravitational field. But this third fundamental trick of Einstein backfired.
It was perfect to get the gravitational equations but it is useless to understand how accelerations work. A crystal clear proof of this is a paradox that is still completely unresolved and Einstein himself admitted this. We regard two balloons filled with unfreezing liquid in empty space somewhere between two galaxies. We spin one of them which becomes distorted. But WHY?
Both balloons can regard as the other is spinning so what makes the asymmetry?
Einstein said that the Universe must be involved. So should we go back to Mach? I don't think so!
In this section I will mention a case where Einstein did suggest a replacement of acceleration with gravity and I will try to show that it is faulty too. Of course, we can not deny that something deep must be in the Equivalence Principle otherwise it wouldn't have worked to get the equations.
But equivalence as total equivalence is false. In fact, this is obvious from an other angle too.
Namely, that General Relativity had an other even more important trick. This is personally related to Einstein in a very strange way too. Before getting to the concrete work on General Relativity he was in a desperate mood asking his friend Grossmann to help. Einstein realized that coordination is itself an obstacle in using the Equivalence Principle but he imagined pretty well what kind of new coordination is needed. So Grossmann knew at once what could be the solution.
This amazing perfect coincidence was the second and final shift in Einstein's view about mathematics. A much later admission was that the greatest mystery of the Universe is why mathematics is the language of nature. An amazing point to get from his youth.
The first big shift was even stranger because then not an earlier schoolmate but his teacher Minkowski stepped in. His four dimensional view was accepted very reluctantly by Einstein.
But now something deeper had to be accepted in the four dimensional space of events which is present in lower dimensions already. We might think that thus a three dimensional simpler example is the key to understand it. But strangely, it has to be two dimensional. The reason is simply that such new coordination is an inner coordination describing part of the perfect one visible only in a wider space. So actually Minkowski's four dimensional space can be imagined as sitting in an even higher dimension but we experience only what we see inside this four.
Can we say things that still decide the motions? Yes we can. And so a simple example is a surface sitting in our real three dimensional space but the ants without stepping into the three dimensional space can establish a lot about their world, the surface.
The true root is Gauss' Theorema Egregium discovered after he attacked some real surveying problems of real surfaces. I will return to this root later.
But now we return to translating observers that thus go also with fix speeds relative to each other.
We avoid the common initiation restriction but this is not the important generalization yet.
The trip that our light rod made from O to P in the $[\ ,\ ]$ system can be imagined to be made from any P_1 point to a P_2 with fix speed and can also be imagined as any real traveller with a watch on his wrist. By the self time formula the τ time interval that his watch advances under the trip is less than $t_2 - t_1$ if these were the departure and arrival times at P_1 and P_2.
The conventional wording is that our traveller "translated" from event $[P_1, t_1]$ to event $[P_2, t_1]$.
The word translate sounds artificial but it's okay because translation is indeed a very artificial trip.
The word event is more important. We already used it but not with the proper respect. Namely, just like now as a simple synonym of a given place and time. This is fundamentally wrong!
Event is actually the most basic concept of Physics but I didn't want to be a Formalist moron to introduce it as such. Now however its real importance comes out. So we can be quite informal and thus use "event" in its very informal sense like a "happening" say a rock concert somewhere.
A fan would go from one to an other for his favorite band. A deep stuff comes out at once.
The second concert is a destination for the fan and he can only reach it if he has not only the place but the time at that place too. So it seems as if the earlier mentioned barbaric replacement with place and time would be unavoidable.

If the fan is a real fan then of course his real destination is the start of the concert. And many times a concert starts indeed later than planned. This is the real event and so it's not a deeper meaning of the event that is the real problem rather that our poor fan has to "translate" there.
Keep fix speed and arrive exactly at the start of the concert.
Luckily, now we'll correct this nonsense and regard any trips from one event to an other.
So now our fan can be a real person who gets there early and sits until the concert starts.
Because such staying in one place is regarded as part of a trip too.
The amazing fact is that for all these possible trips from $[P_1, t_1]$ to $[P_2, t_2]$ the τ_d self time of a real traveller can only be more than the unrealistic translating self time τ . In fact:
The more detour one makes under his trip, the less time he must spend under the travel.
This sounds quite contradictory but only because we usually only mean a place as destination.
Here the arrival time is restricted too! Not actually as a preset time, rather as part of an event.
So this starts to reveal the deeper meaning of events. The full meaning condensed is this:
While in three dimensional space the straight line is also the shortest distance, in the four dimensional world of events the straight line which is the translation is the longest trip.
So $\tau_d < \tau < t_2 - t_1 = \gamma \tau$ is true where τ_d is the self time of a detouring traveller.
Most amazingly, there is no limit in space how far one can detour before the next concert!
But now let's regard a particular version of translations and detours.
Just as staying idle is part of a trip the destination place can also be the starting place. So $P_1 = P_2$.
For such detour we would say it is a return trip but for the staying one observe that time is ticking "always ahead or away". And this forward going of time without changing position counts also as a "translation". This biggest mystery of time of course is not even approached by Relativity.
But we at least realized an other mystery that even in a smallest time we can make a return trip to anywhere. Our wrist watch will show less τ_d time than the difference in local time and accordingly, we too will feel less time subjectively and experience less from the vast distances we went through. Strange but not paradoxical! Well, we just revealed the infamous Twin Paradox.
The reason we didn't realize this is that I only emphasized the possible small τ_d time being sufficient for a big detour. But I didn't emphasize that τ_d can be arbitrary small not only because $\tau_d < \tau < t_2 - t_1$ but can be arbitrary small for any given τ too.
Of course, in our special case of $[P, t_1]$ departure and $[P, t_2]$ destination, the elapsed $t_2 - t_1$ time will be identical with τ the "aging of the P place" while our traveller can age any small τ_d .
A magic trick is helped by smoke and mirrors to hide the simple cause and effects, but a paradox can be over emphasized or distorted by restricted circumstances.
The nowadays popular slogans of the Formalists that paradoxes are only in our minds and nature doesn't follow our intuitions are the most dangerous lies. Paradoxes are real and show that nature is totally humanoid. Understanding is the base of not only knowledge but Nature too.
So to understand the Twin Paradox we first must get rid of the sand they threw in our eyes.
This aging at P must be widened. Why shouldn't someone age still slowest but less boringly!
That is, translate from $[P_1, t_1]$ to $[P_2, t_2]$ while his twin makes a detour as two translations.
There are two main scenarios of this and the first is actually a double paradox if we accept the classical returning twin as a paradox already. In this again a twin stays at home but not forever only till he decides to go after the one who left before. Namely, with a bigger speed to reach him.
Who will be older when they see each other? The doubled surprise is that the one who left first.
The explanation is simple! He translated between the events of his leaving and his twin arriving.
While the other spent some time waiting and then chasing. So this was a detour!
And now the third scenario. They both leave with equal but opposite speeds. Then one of them decides to turn around and catch up with other. The result is obvious, the one who didn't turn around made a translation while the other made a detour so he will be the younger.
And this third scenario is the deepest true twin paradox! Because up to the point of the one turning around they are symmetrical. But the full τ_d smaller self time includes the time while they were still symmetrical. Could time reduction only be happening after the decision to turn around?

Not only insane but the actual reduction formula uses the full travelling time in [,].
So there is no gradual reduction or youngening. Moving clocks don't slow down as such!
The bigger picture revealed by this paradox becomes even clearer if we realize that a simpler version of it was inherent in the already emphasized synchronization paradox.
I only emphasized that the middle point was used but not really used as data.
This is nowhere emphasized in the literature but a consequential one is emphasized very much.
I left it intentionally till now. This is the simple observation that for the moving light rod this synchronization is obviously wrong. O of [,] is running away left, while P is approaching from the right and so a middle point's two light signals shouldn't reach them at the same time for sure!
Observe that again the middleness is not even used, what's more neither the speed of light.
Simple geometry dictates that such synchronization is always wrong. And indeed, the classic view is that light travels in the cosmic space and so such synchronization is only valid there while everywhere else we must adjust it with the fairly simply calculable delays.
But most amazingly, beside this exceptionality of the cosmos as a particular inertial system we also must use in the "fairly simply" the universality of time.
This universal simultaneity of the "now" is destroyed by Relativity.
And as I said, this is mentioned plenty times and crystal clearly, almost as a disguise to hide the bigger truth that a wider subjective simultaneity of time intervals is also false.
The symmetrical departing of the two twins are such time intervals. Not relatable to their future time accumulations. The point is simple! As long as they are far from each other there is no contradiction in regarding symmetrical time distortions and thus differing time accumulations.
For the original twin paradox where only one travels, the contradiction of symmetry was also present but the easy way out was that only that one becomes correct which stayed home because he didn't have to switch systems. An easy over explanation even claimed that the time distortion was caused by the turn around. This was soon realized to be wrong of course.
The time interval of the departing is half of the youngening so we imagine a gradual youngening.
Totally wrong but this was not attacked. So the departing twin's view that the other with the whole universe is moving away was also just mentioned as interesting ha ha ha.
Instead of raising the symmetrical departures as a concrete attack on the subjective view of the time intervals, a very different attack was launched that then lead to the worst misunderstanding.
Namely, how continuous video signals could catch the gradual agings become different yet commonly established by both twins.
The final totally screwed up result is the still present Twin Paradox article on Wiki.
Already the first non video feed explanation takes only few sentences and avoids any paradox!
So this tries to explain the age difference in both twins' view with the same end result.
Not surprisingly, it is a false argument! As smoke and mirrors they use time dilation for one while distance contraction for the other. But at the first regarded the other while at the second himself.
As always, they threw the sand in their own eyes and so were happy to see the Paradox disappear:

Earth perspective

The Earth-based mission control reasons about the journey this way: the round trip will take $t = 2d/v$ = 10 years in Earth time (*i.e.* everybody on Earth will be 10 years older when the ship returns). The amount of time as measured on the ship's clocks and the aging of the travelers during their trip will be reduced by the factor

$$\alpha = \sqrt{1 - v^2/c^2}$$

the reciprocal of the Lorentz factor (time dilation). In this case $\alpha = 0.6$ and the travelers will have aged only $0.6 \times 10 = 6$ years when they return.

Travellers' perspective

The ship's crew members also calculate the particulars of their trip from their perspective. They know that the distant star system and the Earth are moving relative to the ship at speed v during the trip. In their rest frame the distance between the Earth and the star system is $\alpha\, d = 0.6 \times 4 = 2.4$ light years (length contraction), for both the outward and return journeys. Each half of the journey takes $\alpha\, d / v =$

2.4 / 0.8 = 3 years, and the round trip takes twice as long (6 years). Their calculations show that they will arrive home having aged 6 years. The travelers' final calculation about their aging is in complete agreement with the calculations of those on Earth, though they experience the trip quite differently from those who stay at home.

But let's leave this nonsense because even weirder nonsense comes next, and this goes back to Einstein himself. He returned to the Twin Paradox and gave a "solution" involving General Relativity. By this gravitation slows down clocks too, in fact more and more as closer they are.
We might get the impression that the strength of gravity is causing the bigger slow down but this stupidity is only said by really bad parrots. Most monkeys stick with involving the distance climbed against the gravitational field. So it's not that on Mount Everest clocks tick faster, rather the trip taking a clock up there will make it tick faster. You might say what's the difference.
I give you an insane scenario to see it! Imagine a homogenous gravitational field.
If you move against the force you won't find a weaker field but your clock is still getting faster.
By the way, there is an easy way to remember why going not against rather falling with gravity causes a slow down of time. Namely, in black holes time stops completely.
Now comes the point how this gravitational time dilation can "resolve" the Twin Paradox.
I said that translation is an insane trip. Well now we get a really insane translation.
So the returning twin can imagine himself at rest during all of his trip as follows:
Everything was okay, the whole universe was going away from me when suddenly a mysterious gravitational force field appeared. Actually I had a premonition about it and so just in the nick of time I could start my engines and thus counter the field. The Earth and my twin were not that lucky and so stopped flying away and actually turned around and started to move toward me.
All this stopped soon but they still flew toward me with fix speed for years. I also knew the sad consequence of such big fields! The farther you go against it the faster you age. I don't know how they got so far against it in that short time but I'm sure they aged incredibly in those few hours.
If it sounded as a mental patient on a witness stand then I achieved my goal.
Sadly he was correct. His twin aged a lot more when they met again.
To prove that he was not crazy, we would need a video of his "episode" and of the "same" few hours on Earth as decades. But this is meaningless!
There is no meaning of this "same" just as there is no meaning of the "now".

Don't think that I'm so stupid to think that Einstein was so stupid to think that his thought experiment really resolved the Twin Paradox. He knew perfectly well that this thought experiment was merely an other strange connection we can project onto reality by our visions.
But reality is mysterious, period. He knew that but didn't know something more important!
Namely, about understanding. Not full understanding, just understanding something concrete.
The full understanding that doesn't exist is our secret desire. Our soul remembers it but our mind can not follow. Our soul we got and if were lucky then it has nothing to do with luck, that's what we are. So the big problem is the others. We find these strangers around us and only our minds can relate to them. Religion denies this and wants us to relate to them as souls.
True or false is immaterial to what I claim. That there is a fundamental connection involving our mind with the soul of others. Behind it probably lies that the soul develops in many lives with minds but this assumption is also irrelevant to the simple solution that Einstein missed.
We must teach! Explain the concrete understandings to stupid people. Simple but very hard.
But the truly important point beyond this seemingly humanitarian assignment is actually a very selfish result. Discovering that nature is understanding. So it's not the others that is the goal!
It's our own experience of the explaining.
Einstein and Gödel gave not a single class in Princeton where they were kept as trophies.
Instead they chattered about America in German. What an immoral setup already.
We could analyze this for hours but I am toward something much deeper.
Two opposite geniuses. A deep and a wide.
Newton, Gauss, Gödel were the wide geniuses and Cantor, Einstein, Turing the deep geniuses.
Six men who all remained in the deepest darkness about the simplest truth about themselves.

$\mathbf{E = m\,c^2}$

Imagine a rubber ball into which we can pump water through a tube. Using an F force to accelerate this ball while we change its mass, we would have the dilemma, how the force is relating to the motion of the ball. The logical is $F(t) = (m\,v)'(t) = m'(t)\,v + m\,v'(t)$.
Indeed, both the increasing speed and mass requires force simultaneously.
Beside, the mv momentum is a fundamental quantity on its own giving the conservation of momentum, translation of the barycentre and so on. So we got something deeper but is it useful beyond? Why would a mass increase by motion? A hasty idea actually leads to a chicken and egg situation. Namely, we could think that some part of a speed relative to something else can be incorporated into the mass and then the incremented momentum or kinetic energy would give information about how relative speeds must be calculated. But of course then the calculated new relative speed again can be incorporated and so on. Not quite stupid idea but we need something more. First of all, momentum is perfect as it is because it is a vector and the whole Newtonian change was based on using vectors. Energy however was not part of basic dynamics.
So though vectors describe how horses pull a wagon, how the hay becomes a force by the horse was a mystery. Nowadays of course we talk about calories and accept this non vector miracle.
In Relativity this miracle got finally its grand recognition.
Actually, the concept of work as force times distance did enter dynamics and was also realized to relate to the non directional energies of heat, pressure and so on. This messy state was improved by regarding systems of fast moving random particles but nobody expected that something that simple as $E = mc^2$ could give the total energy. And the miracle is still the same as at the origin that a few kilogram of hay can make the horse turn it into any directional force. In fact, now this same miracle is reversed. Moving the parts of an object, this increases its mass in all directions.
This makes sense to say because the most trivial appearance of mass is resistance to be accelerated in any direction. Once it moves it has a hitting capability again proportional with its mass.
They call these two together as inertia and the big recognition of Newton was that a third appearance of mass is causing proportional gravitational force. One needs a bit of basic rules of forces in general too, but the simple cause of what Galileo realized “the law of common fall” is that a heavier object simply means bigger gravitational force but also bigger resistance against this same force and so the end result is a same acceleration of all dropped objects.
The fact that forces are directional but this total E energy is a more basic non directional total, already shows that a non force interpretation of gravity had to exist too.
All very interesting but no clue yet why an increasing mass should have any meaning.
The only logical explanation is what we used already twice, the perpendicular independence.
So a mass moving in a direction can only be losing this exceptional direction if it increases its resistance perpendicularly too as apparent mass change. So let’s start our calculations.

If $m(v)$ denotes the changing mass depending on speed and $E(v) = p\,m(v)$ is its total energy with a p proportionality constant, then the $E(v) - E(0)$ change of this by being accelerated from 0 to v, would be the work invested and calculated as the integral of force with distance as variable. $E(v) - E(0) = pm(v) - pm(0) = \int_{x_0}^{x} F(x)\,dx = \int_{x_0}^{x} (mv)'(t)\,dx = \int_{x_0}^{x} \frac{d(m\,v)}{d(t)}\,dx =$

$$= \int_0^v \frac{d(m\,v)}{d\,v}\,\frac{d\,x}{d\,t}\,d\,v = \int_0^v (mv)'(v)\;x'(t)\;d\,v = \int_0^v [m'(v)\,v + v'(v)\,m(v)]\,v\,dv =$$

$$= \int_0^v [m'(v)\,v + m(v)]\,v\,dv = \int_0^v [m'(v)\,v^2 + m(v)\,v\,]\,dv$$

We applied changes in the variables and their differences.
Then used the simple rule that $(f\,g)' = f'g + g'f$, plus that $x'(t) = v$ and $v'(v) = 1$.

Now we can reverse the integration to derivation and so:

$[\, pm(v) - pm(0)\,]' = pm'(v) = m'(v)\, v^2 + m(v)\, v \;\; / - m'(v)\, v^2 \;\; , \; : (p - v^2)$

Getting: $m'(v) = \frac{v}{p - v^2}\, m(v)$. The solution of this is: $m(v) = q\, (p - v^2)^{-\frac{1}{2}}$.

Thus, a new constant q beside the already used p is allowed.

To verify our solution we'll apply the universal law of derivatives for exponents: $(x^r)' = r\, x^{r-1}$ plus that if an $f(x)$ is used in place of x, we must multiply with $f'(x)$. Then indeed : $m'(v) =$

$[\, q\, (p - v^2)^{-\frac{1}{2}}\,]'(v) = -\frac{1}{2}\, q\, (p - v^2)^{-1\frac{1}{2}} (-2\, v) = v\, (p - v^2)^{-1}\, q\, (p - v^2)^{-\frac{1}{2}} = \frac{v}{p - v^2}\, m(v).$

Now q can be established from the $m(0) = m$ rest mass:

$m(0) = q\, (p - 0)^{-\frac{1}{2}} = q\, \frac{1}{\sqrt{p}} = m$ so $q = m\sqrt{p}$ and $m(v) = m\sqrt{p}\, (p - v^2)^{-\frac{1}{2}} = \frac{m}{\sqrt{1 - \frac{v^2}{p}}}$.

To establish p, we have to make a new assumption.

$\sqrt{1 - \frac{v^2}{p}}$ becomes 0 only if $v^2 = p$. This then means $m(v) = \infty$.

But at what speed should this meaninglessness occur? Not surprisingly at c and so $p = c^2$.
Thus $E(v) = pm(v) = m(v)\, c^2$. Most important is at $v = 0$ meaning $E(0) = m\, c^2$.
So amazingly, we achieved something that has nothing to do with relativity as such.

But now comes the topping on the cake, the final formula for the changing mass as:

$m(v) = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\, m$. So we obtained the γ light dilation factor again as mass "dilation".

Light can't and doesn't have to be accelerated to its c speed. It has no possible momentum change and average work that would lead to the old $m\frac{v^2}{2}$ formula approximating its kinetic energy. But its momentum is $m(c)\, c$ and its energy is $E(c) = (m(c)\, c)\, c = m(c)\, c^2$.
Both of these were actually known and obtained by quantum considerations.
So the photon also suggested the $E(v) = m(v)\, c^2$ formula. Other radiations also carry mass and originally that was the emphasis. Einstein even wrote the equation as $m = \frac{E}{c^2}$ giving the surprising mass that means both inertia and gravity for a radiating energy.

A first experimental verification of General Relativity came through the light's mass too.
But light having particles and they having mass was assumed way before.
This already implies that the lights of stars are bent by our Sun. Not observable of course daytime since we don't see the stars at all. A lucky exemption is an eclipse. How much gravitational bend happens is not trivial though and the first calculation of Einstein was wrong but luckily the first eclipse was not usable either due to bad weather. By the second time Einstein fixed the error and his prediction of the bending angle was perfectly measured.
By the way, the early assumption of photons and they having mass led as its most amazing consequence to realizing the possibility of black holes, by father Michell. A black hole is an object from which the escape velocity is more than c. Amazingly, such can be arbitrary small because not just the mass but the radius of the object counts too. Surprising but remember that Gravity gets larger as objects get closer. A very basic paradox of this is asking why two apples touching can be separated. If touching is zero distance then their gravitational attraction should be infinity.
Well, they don't have actually zero distance from each other since they are not point masses that we use in the gravity formula. Their centers are a few centimeters away.

For a non point mass to calculate the real distance we must use integration, so imagine the object from infinite many little parts. It took Newton months to derive that the Earth's gravitational center is actually its geometrical center. So an apple regarded as point like and also ignoring the terrain, we should use the Earth's radius as distance to calculate the weight of the apple.
By the way, to make the Earth have so incredible gravity on its surface that the escape velocity is c , that is turn it into a black hole, we would have to compress it into the size of a cherry. But even just compressing it to the size of the Moon we could not escape with conventional rockets.

Returning to the more important $E(0) = m c^2$ non relativistic energy content for rest mass, it has strange concrete appearances too if we regard what masses could make up m itself.
So actually we apply inside m again the combinings by energies rather than by the naïve masses.
Quite simply put then $1 + 1 = 2$ is not true anymore or rather $m_1 + m_2 = m$ still allows that:
$m_1(0) + m_2(0) \neq m$. In fact, both $<$ and $>$ can happen so such total of the individual rest masses can be less or more than the full m. The reason is that the parts move, which normally would just suggest that some extra mass in m comes from this energy.
So if m_1 and m_2 are connected by a spring and resonate then the total m will be more.
But things are more complicated because the two masses can cause force fields. If they attract each other then being together means less energy than being separated. So two magnets stuck together weighs not more rather less than weighed separately and added. Similarly a hydrogen atom weighs less than the added masses of a proton and an electron.
Actually the same is true for the Earth and Moon. Here this hidden energy of the combined state is the clearest because by gravity things can get together completely. The proton and electron wouldn't get together due to even deeper hidden energies. The moon and Earth if were combined we would have to separate them to measure their separate rest masses. But here an other interesting possible complication is that such togetherness can be gradual as orbiting.
Then this required separation energy is paradoxical in itself.
A satellite rocket trying to speed up by using its thruster to go forward, will actually slow down by getting to a bigger orbit, that is being more separated. But even here we have hidden further issues because the used energy came from the system as rocket fuel. So $E = m c^2$ is not just the simplest and grandest law but actually the most complicatedly appearing one.
We should make a short history of how the bad energy concepts got cleared up:

Burning a campfire suggested to ancient people that the wood turned into heat.
Then they realized that placing a burning a wood under a closed glass dome on a scale, the scale will not change even after it cools down. Whether the heat was locked in or not became a new problem but it was certain that the carbon atoms of the wood are all there in our glass dome in the carbon dioxide molecules. So the wood literally went up the air!
A modern day version of the campfire is an automobile. Here too if we collect the exhaust materials in a balloon then it will weigh the same as the used up petrol. So again, the petrol didn't turn into the work we created. These facts though do establish the conservation of mass, seem to say that heat or work exists parallel as mere side appearance of matter changes.
The $E = m c^2$ law offers an easy solution to this massless parallel energy mystery as follows:
Some of the wood did turn into heat and some of the petrol did turn into the work done by the car.
It is simply the huge value of c^2 that makes these E energies original $\frac{E}{c^2}$ masses immeasurably small. So the exhaust balloon actually weighed a bit less but we couldn't measure this loss.
Unfortunately, this convenient use of $E = m c^2$ as a mere quantity equation is completely false.
We must involve the seemingly illogical assumptions too that heat has mass and even that a sped up mass is increasing. So a car keeps the "lost" tiny mass as its own mass expansion.
When it slows, so loses speed, it gives this mass too with the corresponding energy to its surrounding. Partially as heat, that carried mass too.
So conversion never happens, mass and energy are both present all along and in all transfers.