

Open Letter To Terence Tao

It has been more than a decade that I approached a mathematician.

The last one was Rod Downey about Randomness, that was my secret obsession in high school. The first ones I approached were physicists a year after my obsession from randomness shifted to an extension of the real numbers but not outward like the complex numbers rather inside the points which allows a new epsilon. The physicality of this seemed so deep to me that I was sure it has to offer some new roads to Quantum Mechanics.

Only Werner Heisenberg replied but he within a week.

Back then in 1965 when I was sixteen years old and spoke not a word English nor German nor Russian, I was very isolated. There wasn't even a proper calculus book in Hungarian.

In New Math only two books existed. Alexandrov's Set Theory and Rozsa Peter's Playing With Infinity written in 1938.

After I got into an opening math high school accidentally, simply because there weren't enough applicants, a miracle happened at once and I got a highest grade on the first day by mistake.

My teacher wanted to show how everybody can miss simple things and told that a Hungarian professor when visited the elementary school Olympics in Moscow was asked what is the result of the expression $(x - a)(x - b)(x - c) \dots$ for all a, b, c, \dots letters of the alphabet.

I had no idea about algebraic expressions, distributive law or anything. In fact, I fell into a double mistake. I thought that a product is only becoming 0 if the last member is 0 and so a wishful second mistake was that x is the last letter of the alphabet making the result 0.

So my hand was up and I was praised. But this made me study so hard that half year later I actually won the mathematical journal competition. Then twice the national competitions and so I was accepted to the math faculty without entry exam. The four years in high school as I said was total isolation. I remember once at a math society meeting, Laszlo Kalmar and Rozsa Peter were sitting next to me. Kalmar's Set Theory text book had a few pages long proof of the Well Ordering Theorem that was a jungle to me and I remember I wished I could talk to him but was too afraid to approach him. By uni the attraction toward new math was even deeper but the Foundations Of Mathematics was only subject at final year. I got lucky again because an unknown calculus lecturer continued the role of my beloved high school teacher. He knew how stupid it would be to teach limits without quantors but he didn't tell this to us. Just wrote on the blackboard a line from an old Hungarian song: "Every woman has a moment in her life when she wants to do what's not right". After the giggling stopped he said: unless we can tell the opposite of this sentence we can never understand calculus.

In spite of my deep depression, only an impetus from an old high school mate made me to leave Hungary. After a train trip with goats and crossing the stone walls built by the partisans of Yugoslavia back in the second world war, we arrived in Italy. In Rome, waiting for my US visa I spent all my time with reading Cohen's "Set Theory And The Continuum Hypothesis".

I had also visions about forcing earlier in high school and when I found out that this was actually done I decided that I have to meet Paul Cohen. I also found out about Non Standard Analysis but still didn't know about Martin Löf's randomness definition and I think Solovay's didn't even exist at this time. This form was what I envisioned in high school.

Two months after I arrived to my "assigned" place in Cleveland, I was already in Stanford.

To get there I was washing dishes in a San Francisco Restaurant and I was very happy!

Cohen was in England so I had to wait and I started to work in the Math Library.

I sadly read Robinson's epilogue to physicists about the potential use of his new numbers.

But I spent most of my time with Shoenfield's Mathematical Logic. I threw it to the wall several times but always returned to it. Now that the Well Ordering Theorem shrank from pages to a line, I knew that those earlier detours in the long proofs are only avoidable by not seeing the bigger pictures. But classical mathematics was still a despised relic to me. Professor Polya walked with me a few times in the university park and told me not to be afraid to talk to Cohen when he returns. The crucial meeting seemed very unsuccessful! I knew I knew something but I couldn't tell what this was. Quite amazingly, Cohen recognized this depth better than I myself.

After I attacked him that turning toward the Riemann Hypothesis is an insanity and he is an idiot if he finishes his thinking about the Continuum Hypothesis, he kicked me out. But next day he came to the Library, called me out and said that he thought a lot about what I said.

In fact, his words were: "Don't think that my vanity, that you blamed for my stupidity is the reason that I came now. So it's not that you drew a parallel between Einstein and me, rather its truth that makes me say that you are probably right and I am wrong. Yet I must think about my things. Maybe it is indeed vanity that drives me. But you definitely should stick to your wider search for understanding." I was surprised, especially that he used the word "understanding" which I didn't remember uttering at our conversation the day before.

I had to find a proper job and my instant "luck" almost became my life's biggest trap. I became the mathematician for a company branching out to computers as many tried at this time in 1970. I wrote the program library which forced me into not just classical but God forbid applied math. As usual, the company (Cintra Physics International) went bankrupt and Tektronix in Oregon bought us. I was one of the three people kept. Our electrical designer who didn't have a high school diploma and wasn't kept, went to Texas Instrument with the stolen blueprints that became the first electronic calculator using integrated circuits just developed there.

My new trap in Portland didn't last long. One morning as I glanced at the thousands of oscilloscope tubes, I had the following vision:

A secretary who types her letter on the IBM electric typewriter looks at the "monitor" and corrects the typos from the screen which then goes page by page to the memory and gets transmitted through the telex line. In Chicago the letter again arrives first on a "monitor".

I was ecstatic that this advantage of Tektronix having already the screen manufacturing can coincide with such an obvious advantage of having a "monitor". This could have been the edge for Tektronix and the end for me but this wasn't meant to be. Luckily my boss being a narrow minded asshole said that my idea is stupid because a hard copy is the main thing. Letter writing and programming on screen is not enough to release a computer with monitor.

I was angry and happy at the same time. I resigned at once and started to pack. A new friend of mine who was already a mathematician living there begged me to reconsider. He said he knew a group of young high school students in Seattle who are designing small computers. He even set a meeting with two of them and wrote their names for me on a paper. One of them was Bill Gates but I never met them and left in three days.

I started to tutor in L.A. again, which I did for the one year I spent at uni in Hungary too.

Here again it was for money but the strange "side effects" continued too. Already back in Hungary sometimes for hours I thought about why the students didn't get something and I always realized that it was my fault and actually the obstacles in understanding are big errors in explanations. My nose for "fishiness" was well known at uni too. When my hand was up, everybody knew I will say "I don't get it". And almost always the triviality turned out to be either not even true or hiding stuff that was the most important. These two, the hardheadedness in following the arbitrary jumps of proofs and the realizing of how the seemingly non math oriented people can cope with exactly the same level of understanding, did not meet yet.

The well known joke is that after the tutor explains something several times and the pupil still doesn't get it, comes a point when after a particularly detailed explanation the tutor says:

How the hell can you still not get it when now I finally understand it.

This joke only emphasizes one side though. Namely, that more detailed thinking about the seemingly logical steps always brings up new things to see. But the crucial point is that the joke is false! Because the pupil always will get everything too. They will, if we are honest.

I returned to Hungary but fifteen years later left again and now live in Australia.

I can not give a detailed record of how my obsession with didactics altered in time.

For decades now I believe that there will come a Didactical Logic that supersedes present Mathematical Logic. But at the same time I believe in a trio of concepts, namely the Road, the Gardens and the Map. These two belief system did not relate up until last week. And this change is the actual cause of why I am writing these lines.

“Roads” are an absolute necessary collection of abilities that can not be marked or evaluated. My article about this is titled “The Seventh Wonder”. It could also be called “The Seven R-s”. The first wonder is “walking” and many toddlers toddle longer. Yet on the streets we only see walking people. We all acquire walking with the same perfection. Then the second wonder “talking” is even more amazing because some kids start much later. And yet again, we all acquire this even more important “Road” to understanding. The third wonder is “reading” the first “R” of the well known three, reading, writing, arithmetic. This naming is actually faulty and not because the funny part of calling “writing” and “arithmetic” an “R”, rather because it shoves something very important fourth under the carpet. Fast tracking reading and writing about which I could tell a lot though, counting is the fifth wonder that brings about a mysterious instant ability not just to count but count in steps both forward and backwards. There is no more beautiful proof than this for how the infinity of naturals is a God given a priory ability. But this natural geometry of addition is not true for multiplication. At the most elementary level this means that the “times table” is a non visual sixth wonder. The “stupidly” memorized times table has hidden effects on our thinking. We can verify or derive that $6 \times 7 = 42$ from additions but to rely on these is a big mistake! And this mistake was committed in the seventies when the so called complex method wanted to avoid the times table. I was there to see the consequences! Though all kids indeed can discover all multiplication values themselves, for some the automatic knowledge follows while for some it never finishes and the discovery process actually becomes an obstacle. So kids must learn the times table! All kids! Both the smart ones who would automatize the discoveries and the stupid ones who wouldn’t. Simply because this was the biggest stupidity to say. The “stupid ones” who wouldn’t “automatize” are actually the really smart ones. Because numeracy is not the true base of mathematics. It is Grammar! And yet we have to force these deeper grammar talents to learn the times table too.

At these early times, in grade one and two an amazing garden can be discovered if the English teacher and the math teacher work together. This is the true formalization of Grammar, into “Grammatics”. But now I skip this revolutionary introduction of the quantors too, because it’s not a “road”, it’s not an unavoidable must.

The seventh wonder is what the English speaking world calls “word problems”.

But we who lived in the “east” as children, can just call it what it is “Larichev”.

Amazingly, I never used this miracle road that leads everyone into mathematics. Never used it for myself, because I entered a math school and so I entered math through Geometry.

This was also the old suggested way. Plato wrote above the entrance to his new Academy that no one should enter who can not master Geometry. But times do change and to accept this is sometimes the hardest, especially when the old way is our own old way too.

And so I did see the word problems help hundreds of students enter the world of mathematics.

But it took me decades to really see why the word problems are such a miraculous road.

The use of variables combined with the algebraic manipulations, the fractions, the equation solving through balancing and the final realization that these calculations coincide with the initial reality. These form a very complex maze subconsciously yet are very simple as required ability. To master it, is a closed minimal set that explodes into understanding many seemingly unrelated fields. Geometrical constructions and other interesting “roads” are not “Roads”. They do not work on their own. This does! The algebraization of the world is a fact we can not ignore. So when Stalin ordered the “perfect” math book for elementary schools, it took a lot of guts from Larichev to create his step by step problem book. In Hungarian it was titled as Collection Of Algebraic Problems. So “what’s the catch” you may ask. And there is one. Roads are abilities! So this word problem solving also should be an acquired ability. Not difficulty of the problems is that matter, rather the easy automated way to solve them Then it is just like walking, talking, reading, writing, counting and multiplying. No marks, no levels of possessing it.

Externally, looking materialistically, every ability has its limitations! Some word problems are much harder you may say. But just the easy ones are already an infinity of cases. Multiplication is not required either for two digit numbers but if some kids can do it good luck with it!

Already counting is a potential infinity but we won’t ask kids to count up to millions.

This magical Road is not my invention and I wasn't even the first to stand up for it! I knew that it exists and used it for hundreds of my students that turned from math haters to math lovers. Because seeing our own success is the only real motivator. The field of infinity underneath is the hidden extra objective filling that makes the emotion of success extra too. But that was all! My private belief and a belief that one day it will be recognized too. Then I saw on the net poor Andrei Toom's articles who moved from Russia to South America and experienced the same success through word problems as anybody who tried it. The Americans reacted negatively of course. Why should we require something from everybody? We are the land of the free. Yes, you can finish high school without being able to add two fractions but we are still the best of the world in maths too. So who cares? Much more sadly he gave lectures in Sweden too and no true acknowledgement happened. The coin just didn't drop because the crucial ability level as transformation igniter was never emphasized by him and the drastic success wasn't tested either. That much is enough about this subject because it's really just a matter of empirical facts. Those who don't want to test this instant door to stop the hatred of elementary school math by the kids, will only change their minds when all else fails. Far away in the future. Until then they will use triangles and squares instead of x and y , they will say "the set of natural numbers" instead of just saying "the natural numbers", and so on. All kinds of phony abstractions enter the school system continually that will just make my point ever more relevant. The explosion will come because the situation gets worse and worse! Of course, there are always the "math talents" who get hooked on this unpopular field that all the hip kids hate. This spreads even more the false belief that some extra talent lies there at all. So in a sense then the talents start to believe in this myth too. The year above me at uni in Budapest contained the four most talented kids that ever came from the University's own special high school. The most promising was Lajos Posa who seemed to me strange already when I saw him in the few problem solving meetings. He stayed in Hungary and got involved in the competitions and education of talented kids. Erdős who discovered him regarded all this as the biggest waste of such talent. This came up as a side point in the few letters I exchanged with Erdős. My main point was how immoral it is that young talents still have to search in vain for good books in Hungarian when Hungary had all the famous mathematicians. Of course this was before the internet. And one would think that the net changed everything for the best. Books are obsolete, all truths spread like wildfire. What's more, computer users have a natural affinity toward math. Adult "nerds" are math likers not math haters. Plus the large amount of I.T. students should naturally turn toward new math, computability. So not just a wave of new interest in classical math but a new look at math as metamathematics should emerge too. And formally, all this is true! Gödel, Turing became household names. Yet even the most elementary facts of both classical and new math remain inaccessible. How can this be? Well the mechanism is quite simple. The devil's name is Wikipedia. Democracy at work, to enslave everybody. The excuse is the easiest part: We are not educators! We are an encyclopedia. So the most important point is to reference your source. If one follows that wild goose chase he'll just fly into insane cycles that never end. But even if you read on, waiting patiently to be explained what the hell is they talk about, you get dizzy because everything has all its sidelines mentioned. It's like to be so general and so exact that no one knows what they talk about. Except those that already know, who write the articles themselves. So the simple fact of prime factorization will tell you about strange number systems Bezout Theorem but an actual simple proof will not be found! That's a fact! In Posa's 1999 book for talented kids, he says that the unique prime factorization is beyond this book. What would he say to my article? I have Didactical Gems on my web site cognum.org especially to prove that things can be explained simply and perfectly if one strives toward this. But Wikipedia is not striving to spread understanding to start with. And Wikipedia is not the single form of the Devil. Blogs are an even better disguise to lie. Every communication that is not intending to make something understood is a lie.

Name droppings are the easiest way to avoid making something understood right there to the actual listeners. And yet the listeners like this. Learning new names. Because most sadly, they don't really want to understand anyway. Just to be better informed! Or just chit chat!

I remember the first time I saw your name was in a blog where you mentioned the Haar Measure in a conversation about the prisoners paradox. I said that guy is crazy. He knows that what he says is way above the head of the listeners but he uses his knowledge to say what?

That he knows what the Haar Measure is? And yet now I write to You.

Today something very strange happened!

My son was watching TV in the other room. Two programs simultaneously as usual.

One was "Brother Sun, Sister Moon" about Saint Francis of Assisi and the other a Star Trek episode about how Data creates a daughter, who sadly dies at the end.

In the last few weeks he refreshed a lot of his math knowledge by talking to me.

He is a film maker and was editing all week but recently talked a lot about the high school days and how I tutored him with great enthusiasm but not pushing him toward math at all.

Few days ago I added a little end section to the article : "Infinity Of Primes" and I changed it a few times. I went back to a bigger article called Primes in the Classical Math section and this had a last page titled Gap Conjectures. I was reading this and was lost. I myself didn't get the situation that I tried to tell here earlier. I committed a grave didactical error.

This became striking because it was all about conjectures or theorems without proofs.

Cramer's Conjecture claims that the prime gaps are all bigger than the square of the limit:

$$p > 7 \quad \rightarrow \quad p^* - p < (\log(p))^2 .$$

Where p^* is the next prime after p and \log is the natural logarithm which as I said is the limit of the gaps. This part of course is an earlier subject the Prime Number Theorem.

The immediate problem was that from Cramer's Conjecture I very nicely showed:

$$p \geq 127 \quad \rightarrow \quad p^* - p < \sqrt{p}$$

Then I came to Hoheisel's result and the apparent limitation of the half power as gap bound.

So we only can prove for some powers above the half value, that from a point the gaps are all under this power. But the method I used at the start to show the \sqrt{p} bounding of the gaps from Cramer's Conjecture is working for any $\sqrt[k]{p}$ roots.

It simply relies on that $(\log(p))^2$ will be overtaken by any $\sqrt[k]{p}$.

Indeed, $(\log(p))^2 < \sqrt[k]{p}$ with $\log(p) = x$ becomes $x^{2k} < e^x$ and the $y = e^x$ function will cross $y = x^{2k}$. In fact, a trivial first crossing is between 1 and 2.

For us the point though is the second crossing from which e^x stays bigger for ever.

Thus, we just have to check p up to this N point and check from which P will we have:

$$p^* - p < \sqrt[k]{p} .$$

Then the $p > N$ cases will be true too by simply $(\log(p))^2 < \sqrt[k]{p}$.

If we assume Cramer's Conjecture. Not showing this for the general k roots was a didactical mistake, making it seem as if the square root were a mystical barrier here too.

What's more, though these "better" $\sqrt[k]{p}$ bounds all follow from Cramer's Conjecture, the higher is k , the second crossing by the exponential function is much later, so N is much bigger and we'll have much more cases to check empirically. This is an amazingly deep and interesting situation. Yet revolves around Cramer's Conjecture and so seemingly lies outside the strictly proved results. Also observe that if we want to improve the square root to a higher root, the result remains the same as quantified statement. Namely that "there is a P value from where $\sqrt[k]{p}$ is a bound on the prime gaps". But in this existence lies a deeper problem because unlike N that is totally simple to calculate, this P is much harder to tell as function of k .

I went into all this in detail because this is a perfect example of what can be seen without actual proofs yet. The sad part was that I didn't show this situation just went for the square root.

All this sent me on a thinking about Maps in general and made me realize what an idiot I was neglecting the complexity of Maps up until now.

Maps are not just drawers with names like algebra, geometry and so on.

Maps contain the exact language of Mathematical Logic and more!

All didactical errors are two fold. Not being exact about what we say but also not daring to say things that we can not quantify yet. To tell what we see, to be honest, we have to talk in Maps.

But vision has been banished from mathematics for a long time now. In fact, it has always been banished. Euclid already, while struggling to formulate his Parallelity Axiom, shoved under the carpet the simple visual fact that parallelity has three very simple subjective forms in a plane.

Fix distance between the two lines, having same angles to crossing third ones and finally simply not crossing ever. The simple Map statement is that these three are implying each other. But the labyrinth of proving this, covers up this simple grand truth. The real mystery to me is how already Euclid was unable or why he was unwilling to convey the simple subjective situation.

Gauss “the fox” of course was not just the greatest classical mathematician but the best to cover up his tracks from which this nickname originates too. The fox does this with its tail by being biologically programmed. We should rise above our boundaries to indirectly lie. But probably lying is a very deep part of being human and that’s why this is so hard.

The obsession with proofs and the names who did the proofs poisons everything.

Erdős admitted this narrow minded philosophy unashamedly. Proof is everything, that’s the only real mathematics! I have the letters with these admissions and there will come a time when these will make him look like an idiot. But that I myself, the big champion of Didactics could be this stupid to regard the Maps as mere guiding fairy tales for visitors from humanity departments or artists who are proud to have hated math all their life, is shocking.

This means that I fell under the spell of Erdős and still regarded proofs as the only real math.

Maps must have the exactness to say what we know or not. Where we come from and where we are going. This means depth and structure. We can talk about Math intelligently without proofs and still say a lot. Or just blubber, drop names and even prove some unimportant side facts with a magicians smile. Just to drive the eyes of the ball.

But a very important distinction is to be drawn between the two kinds of lies that we experience. This duality exists outside mathematics too and I start with a grand claim.

The lies in social realities versus the lies in the media is an interrelating jungle.

In mathematics, the corresponding duality is the lies by the professionals who prove theorems and the lies in education. And amazingly there is no jungle here. Education has no feedback.

That’s why education is unvalued, unappreciated as having any serious merit. We have Nobel peace prize but spreading knowledge which is the greatest tool towards peace is not recognized.

Of course Nobel was a narrow minded math hater who intentionally wanted to “punish” all mathematicians and education wasn’t even in his mind generally. The fact that now we have economic Nobel prizes for a sub class of lesser mathematicians is the joke of history.

So though history is a lie it is also an inconsistent lie.

But this probably wider truth that education is a mere second grade lie that like a dog follows the official “knowledge” is an advantage because we can pinpoint the official lie better.

So the line of Euclid and Gauss not to talk about visions is the simple high level lying that indeed became solid as a rock. This is what Erdős wasn’t afraid to say out loud “proof is everything, that’s the only real mathematics”. I said above, I fell under the spell of Erdős even though I know I had a spark in me from the earliest days of my life that works against this spell.

And the spark is in many! Once they start to write text books the spell takes over but even then a clarity obtained by the honesty of following the visions not just the proofs remains.

I remember that Sierspinsky’s Number Theory was such a refreshing experience for me.

But this is a rarity. The “best” text books are very bad as I mentioned about Shoenfield’s Mathematical Logic. But there were some who didn’t write text books and yet expressed their feelings about this whole affair. Hilbert said that if someone understands something then he can explain that to the first man on the street. I think I am the only person who takes this claim seriously. But we slipped back again to education even though I wanted to show the top lies.

Before I do that I have to reveal a major distinction between the didactics of classical versus new math, which probably will also be important in the first steps toward a Didactical Logic.

I mentioned that already in high school I turned toward new math which then simply meant Set Theory for me and latter widened to Mathematical Logic. And yet the obsession of defining Randomness that came from nowhere lead me toward Effectivity.

Historically, Effectivity started earlier by looking behind Gödel's results and the first application for randomness by Church was only the result of this. I had of course no knowledge of these at all but even when I learnt about Gödel's Incompleteness, I made no connection toward Effectivity myself. I was blind as a bat.

A little book of Rosser helped me to see finally what's going on. That the undecidability of the theorems as a set is the cause of the existence of undecidable statements.

Negation is an artificially created duality of the statements. The derivable statements leave a complement that is much more complex, namely is not generable by any Effective method.

This at once means that there has to be undecidable statement because otherwise the formal negation as an added step after the derivations would give a generation of all the non theorems.

This is the coin that has to drop for anybody who wants to see the truth behind Gödel's result.

The usual blubberings about self asserting statements, liar's paradox are all pushing proof facts in front of the vision. The vision is the set of theorems versus the non theorems. This vision has nothing to do with negation. That's why the Effectivity vision is the wider. The set of the achievable situations on a chessboard leaves a more complex complement set. The set of the unachievable situations. The achievable ones can be verified by their game history, the unachievable ones can only be verified by checking all possible infinite games.

Gödel's original vision that the limitation of the language causes the incompleteness is false.

But it is true that to prove the higher complexity of the set of non theorems, we must use diagonality and therefore the self-asserting statements come in. The really interesting side question is then why only diagonality can provide the proof for something that is so widely true on its own. May it turn out to become provable in less particular ways or diagonality is a barrier in negative proofs? I will return to this in a minute through Rice's Theorem.

The real moral of the story is that Shoenfield's thick book could not show me the simple truth yet Rosser's simple little pamphlet could. The even deeper moral for me is that something blocked me to realize this whole Effectivity vision lying behind the Logical derivabilities.

Even more strangely, then I didn't return to Randomness with this new vision. Only twenty years ago did I start again to go in that direction.

An other fact is that the coin drop for the Gödel Completeness Theorem came also externally and relatively very late. The uselessness of Shoenfield as "vision giver" is even more evident in this respect. His artificially chosen axiomatization of Logic as start makes the Completeness a rabbit pulled out of a badly constructed hat. Again, only a little pamphlet written by Wainer and Wallen helped me to experience the coin drop. Of course, you may say that if you want to go in order you have to start with artificial assumptions. But that is not true. The real moral of the story here is that there is only one single way to start, namely with Tait's system. He never got the proper credit for discovering this magical simplification of Gentzen's monstrosity.

ALL LOGIC BOOKS MUST START WITH TAIT'S SYSTEM.

This system can be extended to all more practical variants of Logic and yet shows completeness as a natural consequence. In fact, there is an even better way to start with the original vision of Hilbert's quantor introductions but not as implicative rules rather as statement set extensions.

Then the Tait system is merely a rule system to use formulas to achieve the same.

The bigger point that I am claiming though is that here in New Math, all didactical roads and visions are obtainable already and the details follow almost automatically too.

There are no Gardens, no detours. Visions can be perfectly detailed to proofs.

And yet though the four pillars, Sets, Logic, Effectivity and Randomness are quite clear now and interrelate intricately, their results do not relate to classical problems.

The fact that halving a number till its odd then trippleing and adding 1 and repeating these always gets back to 1 is either not provable or very hard to prove. To prove which is the case is not the real goal. The real goal is to see why either of these is true.

Even more importantly, why the four pillars of New Math could not approach this. Fermat's Last Theorem is even richer in its arbitrariness because the proofs for the increasing exponents are harder and harder, going deeper and deeper into an unknown jungle. The existence of this jungle is what makes classical mathematics relevant. So unfortunately Kronecker's bitter anti Set Theory sentiments did come true though obviously not the way he meant them. For me who despised classical math as a young man this return to classical math is even more crucial. My belief that visions and Didactics will be at the end of the tunnel is firm and so in a sense the rich and untamed classical math is a hope. What I claimed above was that New Math as a field on its own, regardless its unsuccessfulness for classical math is actually a very successful field because visions tell all its details. And it rules classical math without penetrating it, that is telling the details there. So it's logical to start catching the denial of visions here in New Math. I mentioned Completeness and Incompleteness above but strangely these are not the most distorted facts in new math. I come now to this third most important matter: Above I mentioned the strange role of diagonality. Turing's original article is the most beautiful crystal clear step in shifting diagonality from language to Effectivity. So what I mentioned about Rosser's little book, would probably have happened too if I read Turing's article back then. Maybe its my fault I didn't. Shoenfield never mentions his name either. But now comes a point, where the plot thickens. Kleene's book was a Bible before Shoenfield's and Shoenfield even mentions Kleene's Metamathematics as the most important influence. Diagonality as the singular source of a derivable set with non derivable complement was not the real vision about these non derivable sets. Everybody knew that they are the typical. Whenever a system that generates a set is complex enough, the complement will be non generable. To gather an arsenal of these would have been very nice but did not happen. And yet it was lying there to be seen!!!! Turing did not see it, Kleene did not see it. How???? Why???? Because here vision shows its true priority versus provability. Rice saw this magic even though he had to become a Formalist too and prove something exact. This something sounds only a side issue: If we divide the set of all programs perfectly, that both halves contain all program variants too and this splitting is not the trivial all in one and nothing in the other half, then one of the two sets must be not generable. Seemingly we obtained non generability without diagonality. The proof of course relies on diagonality and so we might even ignore the whole result. In fact, Kleene's famous Recursion Theorem implies the full claim instantly, so praise the Lord, the Bible contains everything. But an amazing postscript to Rice's Theorem shows an arsenal of such divisions of the set of programs where one of the sets is definitely a generated set. For these then of course Rice's Theorem automatically claims the complement to be non generable. So what are these magical, automatically generable program sets. The magic is simply to use the programs. Of course, every text is a program! If it's not then it's simply a stupid program. The mechanical generations of numbers or more generally texts are usually done by calculating things from these as data. But every data can be used as program too. We simply have to run it and wait what it will produce. If we choose some feature that can be recognized within finite time from the running, then collecting all such programs that have this feature will be exactly a set required by Rice's Theorem. So the set of programs that will not show the feature is not generable. We simply can not collect those programs by a single program! So finally, the concept of "program" received its true place in our visions! Collecting programs by their runnings is the secret weapon to get ungenerable complements. Now every sane person would say eureka we made a big breakthrough. Not the mathematicians! Poor Turing just committed suicide, so his lack of enthusiasm is forgivable. But nobody else jumped for joy either. Rice's Theorem entered the text books without any fanfare as merely some side consequence of Kleene's results. But even this infiltration of Rice's Theorem happened only very slowly. Ten years was not enough to secretly take its role in the education. So not surprisingly, Shoenfield doesn't even mention it!!!!!! This is the man who says in the preface that it took decades for him to see that Mathematical Logic is not merely a collection vague results but a consistent tool.

My obsession with Shoenfield's attitude is very important!

He was not an Erdős nor a Terence Tao. These are three different psychological forms of Formalism. If you'll see this, then you'll see the most important point about yourself!

Looking from the Future, Shoenfield was much worse than Erdős! Because the Future with capital F will only judge what one saw and yet didn't see. The depth of blindness.

And yet Shoenfield's Mathematical Logic book is still the most exact and complete in some sense. So if I say that Shoenfield is the Devil then it is not just a deeply emotional and exaggerated negative judgment but an incredible praise too. To publish the facts he so painstakingly collected with the exact derivations is an act of unselfishness. A generation of mathematicians learnt everything from him. So why wasn't he a good man? He was!

Erdős never cared to teach anybody anything. Yet he praised young talents with his own money. So why wasn't he a good man? He was too! Yet he is not even deserved to be called the Devil.

And what about You? Well that's the big question. What will you do after you read this letter.

But before I hammer you further and more concretely about the Wikipedia article, I have to talk about a very important other New Math result. In fact, this is the only one that is comparable in importance to Rice's Theorem that truly introduced the concept of "program". This one, the "Well Ordering Theorem" is much older and so it's neglect is deeper too.

As I mentioned earlier, I was attached to this theorem since high school and I needed forty years to see clearly its meaning. This grand result showed that the visually so obvious exhausting of sets by picking always new elements can be replaced by simply a property collection. So in fact, time can be exorcised from Set Theory and the purely spatial property collection can replace it. I wrote an article about this magical exercise and reading that you will see why the one line proofs for it are lies. But the truth is that Randomness will connect to this and only then will the fault of the one line proof be black and white.

So now I come back to the present when I realized the mentioned error at the gap conjectures.

My old articles are probably full of grave didactical errors but I at least admit these errors.

More importantly, I believe in the importance of correcting them. And most importantly I believe in didactical perfection.

When I corrected the Gap Conjectures page I stepped back to the previous pages about the Prime Number Theorem and started to explain its meaning to my Son.

It was today that he was looking it up on Wikipedia and showed me the monstrosity there.

So actually, we were watching Brother Sun, Sister Moon, Star Trek and the Wikipedia page simultaneously. Seeing your name mentioned there, made me decide to write this letter to you.

But probably seeing Francis going to the Pope was the real motivation.

You are not the Pope but you are famous, your word counts. You could easily alter not just the Wikipedia articles but maybe even the whole public education of mathematics.

The real question is of course whether there is a slight feeling in you that there is something terribly wrong. Francis was taken already outside to be punished when his words sank in and the Pope realized that he was right. Jesus would not like what was happening in his name.

Mathematics is a sanctity too because everybody can understand it and it transforms everybody.

But we live in an age when understanding is not a goal neither by the teachers nor the students.

This bigger problem is outside us but starting from the black and white lies we might initiate a turn for better in general. But again the real question is whether you see the wrong.

Read the Wikipedia article and tell me that it explains its subject the Prime Number Theorem.

If you think so then there is nothing to say for me.

Terence Tao using analogies of music to describe the proof of the Prime Number Theorem is still okay because it is an analogy to avoid the long and tedious job of going into the Garden.

Though for him this is a joy, for some it is a pain in the ass. The story becomes darker when Terence Tao's analogy is used in the Wikipedia article titled Prime Number Theorem.

The article starts with the obligatory abbreviation P.N.T. to ensure the reader that he is smart and already learnt something. You may say I am mean because simply referring to the subject in shorter way can be useful. But then we should tell that the actual name means simply the number of primes in short, so $\pi(n)$ is the prime number function. Instead calling this prime counting function, enforces that this theorem is about the distribution of the prime numbers.

And indeed, they explicitly claim this! And I would go along with this too if they wouldn't fall back to the old obsession of approximating merely $\pi(n)$.

But most of all, before you want to approximate something you should check it out.

The simplest fact you should see about $\pi(n)$ is that the numbers $n!+2, n!+3, \dots, n!+n$ are each dividable by $2, 3, \dots, n$ and so are not primes. Having such arbitrary large number of consecutive numbers without primes means that $\pi(n)$ will remain same valued for arbitrary large increases of n . So to approximate it by some function exactly is a hopeless task.

If you want to be even more precise then you can of course explain that approximation abbreviated as \approx would mean that the actual values are closer and closer as n increases.

A natural idea to alter the big non changing sections of a function is by a division with n , that is, regard $\frac{\pi(n)}{n}$. This loses the monotony of $\pi(n)$ but a beautiful new simplicity comes about:

If n is a new prime $\frac{\pi(n)}{n}$ will increase. If n is not a prime it will decrease.

So we obtained a fluctuating sequence of fractions under 1.

The truly interesting observation is then that the reversed fraction $\frac{n}{\pi(n)}$ is even better because it is the average distance or gap between the primes up to n .

So returning again to the reciprocal value $\frac{\pi(n)}{n}$, it is thus also the probability of a randomly picked number up to n being prime. Both fraction sequence is interesting, the fluctuating experimental primality chance and the changing average gap.

Then the logical problem is this: We know that the primes become less and less but this lessening or raring of the primes shouldn't be measured as their average changes up to an n rather how it is changing around n as n increases. This of course though is more intuitive is also harder to measure because then we have to decide what surroundings we choose.

So here we talk about something that is actually not quantified precisely yet, the local average.

As I said, a Map should talk about things that are not totally exact yet.

In a proof this shouldn't occur but we are explaining the meanings.

Here comes the natural thought that this problem of the two ways of averaging is probably not strictly related to the question of how the prime numbers average in either sense.

An idealized local average knowledge can tell the empirically simpler up to the values average.

We might think that the up to n average is much smaller than the local, around n because this local average is increasing. The early smaller gaps bring down the average.

The big surprise is that actually the difference is only 1.

This value itself is amazing but its lowness is not that strange:

Indeed, up to a million, most numbers are around one million.

Now comes the bigger revelation, that the local average of the gaps around n is $\log(n)$.

Nature's mystery at work! The simplest but seemingly totally unrelated constant, Eulers number can tell this average. We should even pause for a second and see how this yet not even precisely defined local average of the gaps and our claim relate experimentally.

The ten base logarithm for 0 ending ten powers is of course simply this number of the 0-s.

For example $\log_{10} 1000,000 = 6$ because $10^6 = 1000,000$.

The $e = 2.718\dots$ base logarithm can be obtained by multiplying the ten based with $\log(10) \approx 2.3$ and so $\log(1000,000) = 6 \times 2.3 \approx 14$. So this is the average gap around a million.

In other words, about every 14-th number is a prime.

Using our previous revelation of course the average gap up to a million is about 13.

Unlike the local average that is a bit foggy, this average up to an n is always precise empirically as $\frac{n}{\pi(n)}$ so what we claim is that $\frac{n}{\pi(n)} \approx \log(n) - 1$.

All meanings are crystal clear! Average gap of primes up to n is approached by $\log(n) - 1$.

This means closer and closer values as n is larger and larger!

These were the intelligent hindsight of our present knowledge that put the facts straight and doesn't allow us to get lost in historical blubberings.

Now we can mention the obsessions with $\pi(n)$ and the strange results of Legendre and Gauss' reaction to it. But most importantly, we can come to the third revelation that a seemingly much simpler consequence of this claim was enough to prove this approximation. Namely, if two functions that go to infinity but don't approach each other merely their ratio is becoming 1 then this can be regarded as them growing in equal rate and abbreviated as \sim .

Then obviously by our claim $\frac{n}{\pi(n)} \sim \log(n) - 1$ is true too.

In fact, this -1 is irrelevant here so $\frac{n}{\pi(n)} \sim \log(n)$.

Amazingly, this implies the main \approx claim.

Also, this "easier" \sim growth rate law became known as the Prime Number Theorem because we can also write this as : $\pi(n) \sim \frac{n}{\log(n)}$.

So indeed, the prime number or the number of primes is approximated though not exactly only in its "growth tendency".

To intelligently tell a story is not storytelling! It is a Map! To tell the Map of mathematics is what Wikipedia should at least try. Symphonies by Terence Tao is a crime against clarity.

It is using your name to throw sand in our eyes. And you should be upset by this.

That's the real message of my letter. To be the nice guy who is respected and loved by everybody can very easily go into being the silent partner of scumbags.

Those who see hatred in my eyes should look closer. I am the most loving person because I preach a future when everybody can understand. And understanding is the only happiness.

But loving the truth must mean sometimes to strike down on those who speak falsely in public.

Those who do not strive to spread understanding just are very careful not to tell anything false are the biggest liars. Whether they are politicians carefully avoiding the real future, or preachers avoiding the present, or teachers avoiding to transfer visions. And this is the most important!

The cause of bad teaching is irrelevant. Is somebody a bad teacher because himself doesn't see what he explains or just can not explain it well? It's irrelevant! If you can not explain, then support those who can. Our distorted history looks with a smile on that Newton, Gauss, Einstein hated to teach. But more importantly we don't reward clarifications. Not only there are no prizes for breakthroughs in vision. In math we actually deny vision.

This tendency toward bad Formalism is infectious! We all fell back to it with an ease.

To be didactical is hard! Its not just the ability to look at what we say with a beginners eye.

It is repeatedly returning to this ability and care about becoming better and better.

The amazing effects of didactical clarity are only experienced from the beginners' eye lit up and start to love something that they thought could not even approach. These total beginners are not a peer group that gives you confirmation about your status.

Of course, beside this humanistic side is my bigger claim. That Didactics is the future of knowledge itself. This future vision I didn't even go into yet. Once if you are interested I will tell this happier story.