Two Brothers, Imaginary Twins and a Lot of Lies

I haven't been watching much TV lately, but one exception is "2 & A Half Men".
Charlie Sheen in "Ferris Bueller's Day Off" is one of my favourite supporting characters and he still plays himself quite naturally. In this last episode, his real life brother, Emilio Estevez, came on board right at the start. He was supposed to be an old friend with the same life philosophy as Charlie, but it just didn't work, even for the few seconds that he appeared. He didn't age with the same luck as Sheen, so the words coming out of his ballooned head, were very fake and only the "comedy" of the real life brother as opposed to the on screen one could ease the whole phoniness. But as I said, it was only for a few seconds anyway, because strangely he dies right at the beginning and the episode is more about Charlie "thinking" about life.
For me, the scene was about something else. On screen karma is an interesting concept and I should tackle it later, but here the death as punishment was for something that probably nobody noticed around the world, except me. Charlie says to Estevez, "You are a great man.", to which he replied, "No, Albert Einstein was a great man, I am merely a **...** ". It was a bit out of context from his character.
It is almost a compulsory cliché to place Einstein posters in any films or TV shows, where college kids or even just smart kids are portrayed. Usually, the funnier pictures, like the one with his tongue out or ones with the old warm look or violin in his hands, are used. And of course, the $E = mc^2$ is another symbol that appears in the media. There are two reasons that make my reaction against this particular name dropping so bitter and sad. Firstly, because it reminds me how unexplained Einstein's incredible revolution in physics remained. And secondly, because it reflects how false is even the human face that they try to use instead of explaining the new knowledge that Einstein left for us. But both of these, that is the actual physics and the man behind it split in two.
The word, "relativity", is well known and most people who listen more carefully or solve crossword puzzles, even know that there was a special and a general relativity. But these are not the two simple sides I refer to, rather two stages of explaining the same. The two sides are the $E = mc^2$ formula and the twin paradox. The letters in this formula are well known. E stands for energy, m for mass and c is the speed of light. The twin paradox is also well known, claiming that if one of the twins stays on earth, while the other goes away and comes back from high speed travels, then the returning one will age less and will appear younger than the one who stayed on earth. This of course, at once expresses that relativity of time must be the crucial feature of relativity. So just as space is obviously relative, because different observers can be at different places, time is relative too, and doesn't just flow the same way for all observers.
The $E = mc^2$ formula became popular, merely due to its simplicity and so it represents the challenge to shed light behind it, besides the three formal namings of the letters. The twin paradox is quite the opposite, no formula, just a mere claim. Somebody can age slower, just by travelling. Seemingly, here it should be easier to explain how this can happen, but as we'll see this is the harder nut to crack. In fact, it's not crackable at all.
Now the personal lie of the popularizations, that is Einstein himself, also splits, because we can unmask Einstein the scientist, but also the man himself. So we have four subjects:
Einstein the man
Einstein the scientist
The three meanings of $E = mc^2$
The ten layers of the twin paradox

These are intermingled, but basically give the order of exposing them too.
It is almost unbelievable that this level of lie and deception could survive for this long about the greatest scientist of the 20th century. In fact, Einstein has only one equal, Newton. So we should start with some parallels. Indeed, the negatives of Newton's personality are not advertised either. But still, they are pretty much well documented. We can all find the facts that he was eager to belong to the aristocracy. That he was ashamed about his niece's help, to get him ahead. That he died as a virgin. That he died as a rich man, because he never spent his income. That he attended executions of money forgers who were caught by his scientific results as the head of the mint. That he believed the empty claims by some of his colleagues in the royal academy about Leibniz stealing his calculus. Even dancing at the news of Leibniz' death.
One "sick puppy" would sum it up and we are talking about the greatest mind ever.
But here at Einstein, we are dealing with something entirely new. It's not just that the closet secrets came to light too late, but at the same time, an institutional complete lie was created too. And this was not just the obvious Hollywood bullshit, neither an anti-anti-semite conspiracy. In fact, the demagogue and truly anti-semitic attacks against Einstein completely missed their intentions. Simply they were not aware of what Einstein really was as a man. They still not are, so they try to portray him as a thief. Einstein stealing the special relativity from Poincare and the general relativity from Hilbert, is ludicrous but most importantly, avoids the real issues we'll approach in the second subject of Einstein as scientist. So quite unbelievably, the truth, this very recent truth could stay in the vacuum to use an "appropriate" analogy. The first paradox of Einstein the man is the following: He clearly knew that he was the chosen one to solve the riddle of light and matter. He was a rebel, he didn't care about status. He knew that working a patent office can give more freedom to think about the truth, than belonging to a hierarchy of academics. Now all this should mean, that when his fiancé gets pregnant, he's not afraid to face his parents and the world. Indeed, thousands got married after having a child. Yet he sends home the young pregnant woman, in secret, to the Balkans, to have an abortion. He asks in a letter, quite ashamedly, "Did you take care of the thing?". But she decided to bear the girl and give it away. So Einstein's daughter was living in some hell hole family and died at an early age. Of course, they got married later, and the poor woman kept her secret. They divorced and she raised their other two children Einstein gave the full Nobel prize money to his ex-wife to help out. The world portrayed this first wife as a neurotic, to justify the divorce. The lost daughter and keeping it in secret, not to blemish Einstein, portrays a very healthy martyr. If the truth were, that his wife, the mother of his three children, was unstable, that would make his role even more questionable in this whole tragic story. The simple fact is that he didn't care! Now we might think that his preoccupation with physics, understanding the truth in nature, lead to the ignorance towards his family. But this is not the case. The second subject, Einstein as a scientist, reveals that his only concern was finding the truth, to become the chosen one. Or to put it negatively, he didn't give a damn about whether people understood his theory or not. In a sense, there is a positive side, to this narrow minded egocentric vanity. Namely, a very deep idealism, that a pre-existing truth as the mathematical equations of nature, rules everything, so once found it becomes undeniable, overpowering and verifiable. So the human success doesn't have to come from humans, rather from society. And indeed, we still live in this verificational post history of relativity. But Einstein was already beyond that too. He knew that his only equal, Newton, also discovered an undeniable, absolute truth, and yet it turned out to be inaccurate and non final. This of course, in itself is a catch-22. Not because Einstein himself was the

one who nailed the coffin of classical Newtonian physics, rather because Newton was absolute and relativistic physics is not a better physics, even if it is more accurate. But this vision of absolute, being not the accurate mathematical equations, rather the field of understanding, that as side effect creates the equations, was totally beyond his comprehension. So then, the sheer fact that quantum mechanics was still outside the realm of relativity, also meant that a third, even more accurate level has to exist. This uncomfortable imperfection was that drove him to the obsession to find the Unified Grand Theory. To reach the absolute level, to become the best. This didn't happen, he spent decades searching in vain. The real mystery is that this ignorance of the higher importance of understandings, was probably a necessary prequesite to believe in the equations themselves and thus, being able to home in to the final forms. But his ignorance to the pure root of idealism, to understanding as the only eternal entity, lead to the instant karma too, a bitter unsuccessful life. This explains my point better too.
I don't want to bring Einstein down. He suffered enough. My point is against the insane world, that uses his false image as a vacuous idol.
The people he hurt in his professional life meant nothing to him. Just as Newton, the most objective self critic, was able to be deceived to believe that Leibniz was conspiring against him, Einstein too, regarded even those who fought for him in the social arena, as potential threats to his priority as the singular discoverer. Between Newton and Einstein, we can add Gauss as the third tragic genius. The real tragedy is the falsification of history. The weakness to see the drama, while obeying some false respect to the actors. This was allegoric, but with the mass media, the atrocity became actual, because today we are continually hypnotized about "greatness".
Turning to $E = mc^2$, a few months ago there was a British documentary about this particular subject and a really annoying professor from New York university "explained" it by saying that energy and mass are like two sides of a coin. I wonder why Einstein didn't say this instead of special relativity. Most importantly though, there are very simple and common sense meanings in the first quantitative side of this equation. But before I get to these, I want to repeat that Einstein arrived to this formula from his "mission" to unify light and matter. This road is not vital to understand the equation, but helps to put it in a historical line.
Light was already a conflict in Newton's time. Quite naturally, Newton tried to imagine light as a stream of particles. This would have fitted very well into his mechanics. Unfortunately, no special equation was found for these very special bits of matter. On the other hand, imagining them as waves, travelling in a medium, like sound in the air, gave very concrete results about reflections, colors and so on. When finally electricity and magnetism was combined in Maxwell's famous equations, then seemingly the wave picture won by a knockout. The realization that light is merely an electromagnetic wave is the short essence of this knockout. The fact that the sun's light reaches earth, shows that the medium of the electromagnetic waves must be the empty vacuum. Maxwell was a mathematical wiz, just as Kepler was before Newton. Just as Kepler hit upon the right equations of the planets, without proper physics, Maxwell hit upon the right light equations without proper new physics. Quite amazingly, the young Maxwell even wrote an essay on ellipsoids. The analogy is of course a bit loose. While Kepler simply accepted the planetary laws as God's rules, Maxwell tried to furnish the vacuum with magical abilities to propagate light in the way they travel. The most crucial problem with vacuum is not that it is nothing, so it shouldn't have a structure, but that we don't know what system should contain the vacuum properly that is standing. Clearly, the earth is travelling in it, but even the sun, because our galaxy is revolving. And of course, the universe is expanding, though this was not known in Maxwell's time yet. So just to concentrate on the earth's motion, an

experiment was devised, that would measure the speed of light in different directions, relative to the earth's motion. The speed of light is so big that old fashioned methods are unusable. Archimedes once wanted to use mirrors between mountains and failed. Strangely, now again, mirrors were used, but not the delays in time directly were supposed to be measured. Instead, the effect of interference could show even the tiniest changes in time. The results were negative. No change in the travelling, splitting, recombining of lights, regardless what direction these were applied, relative to the earth's motion. They even tried to explain the failure by assuming that the earth is carrying the vacuum's essence, the ether with itself. Alternatively, it was also assumed that maybe the instruments go through some changes, due to the ether itself and these cancel out the changes in speeds. This was the line on which Poincare went ahead and amazingly he used the word relativity and discovered the $E = mc^2$ formula before Einstein. But it was all in "vacuum", in both senses of the word. That is actually with the assumption of the ether, and symbolically as out of any physical reality, by merely assuming the shrinking of objects, just to explain the negative results of changing light speeds.

Einstein used the word relativity in a different sense, more in the line of what we call today, the relativity principle of Galileo. This is the well known fact that if a system travels with fix speed, then we can't feel it. In a train, we can play ping pong, we can pour champagne. The root of this phenomenon is what later Newton called, his first law, that fix speed travel is kept without force. Applying it for a whole laboratory then everything keeps on travelling together. To use it even more elaborately, we can regard perpendicular forces to the fix speed. So, in the train, the champagne pours down normally, because the forward speed of the liquid and the cup is the same, and only the gravitation works, that is perpendicular. Einstein went back to Galileo's observations and took it not as an analytic result like Newton, the weird coincidences of motions, rather as a fundamental principle. So if a box is travelling with fix speed in space, then travelling in the box, we can't establish the motion from inside experiments. In a sense, the motion is not even a reality. Indeed, we only know it, from looking outside, from outer objects. Maybe we don't even move, maybe the outside objects move. Now this would be merely a philosophical stand point, rejecting the analytic explanations of Newton and rather regard it as something much deeper. The crucial new point was that Einstein incorporated light into this relativity of fix speed motions or "translations". Observe, that already the word relativity is totally confusing, because it is actually an absolutity that we claim. Indeed, experiments are absolute, they have the same results in every translating laboratories. The name, "relative", makes only sense if we return to the Newtonian analytic explanation of this absoluteness. Indeed, instead of laboratories, we can use whole infinite coordinate systems that travel relative to each other with fix speeds. In fact, the best to start with the purely mathematical fact, that if we use coordinate geometry to show something, then strangely, the proof would depend on where we place our objects. Or to put it another way, using different coordinate systems we would get different derivations. Now a geometrical fact of course, would only depend on the distances among the objects or points, not to the coordinate lines. Shifting the system for example, the inner distances don't change, because each coordinate values change by same shifts. Turning a system is much more complicated, but still keeps the inner distances. Travelling systems mean continuously changing shifts. Actually, the problem of turning is still not avoided completely, because the initial turned position of the coordinates would still remain. This annoying little problem will come back amazingly gloriously in Einstein's new relativity. But right now we can see that

mathematical changing, that is transformation of the coordinates, is the root of the absoluteness of inner facts. So the coordinate systems are irrelevant, they are relative.
Going from the more physical moving laboratories to the more mathematical relative coordinate systems was also Einstein's formal road. So strangely, seemingly the same analytic path is taken. But this is misleading because the crucial point is that the transformation of coordinate systems is not used to explain how one absolute system's truth can reappear in the others. The travelling train is merely an illusion of a standing system, but for Einstein, all translating systems are absolutely equal, that is relative. But then, the whole inside or outside problem enters with a new emphasis. Obviously, the distinction is sharp. Only the inside experiments, the inner distances are absolute, while the universe outside may seem different. Most importantly, the different systems claim the same differences about each other. This of course, will lead to differences about the common outside universe too. But light that travels in space plays a special role. It's not like a star, that one system can identify to be here or there, and others differently. Light can only be regarded as a potentially interiorizable motion. Light will carry signals of places and times, but the speed of light is not such exterior difference. It is the same for all translating systems. Our subjective vision tries to regard light as a moving point and then different systems should see different speeds, but this is false. Light as travelling point in itself, is non existent. There is no absolute vacuum, especially absolute ether, in which light could be absolute. Light, just like everything else is only experiencable as relative appearance, but its speed is internal to all systems, so must be absolute. But is it possible to go from old fashioned coordinate shiftings and turnings to new transformations, that allow a fix speed at all? As it "turns out", it's not only possible, but in a sense it becomes even simpler in a very abstract sense. Namely, that's why I put the "turn out" in quotation marks, because instead of shiftings and keeping time fix, we'll end up with a single "turn" in four dimensions, that is including time.
The most surprising fact of these new transformations is that the two most obvious old facts, the invariance of the inner distances and the universality of time, are both given up, for the sake of achieving the fix light speed. On the other hand, the old fashioned symmetry of both systems regarding the other the same way, still remains. The most important third feature is that now a new question of physical reality versus observation enters. Indeed, in old fashioned transformations, as I just mentioned above, we usually choose externally, which one is the real and the other only seemed similar. Here now, we don't have such choice, all systems are relative, that is absolute. So then, the question itself, whether the differently observed inner distances and time intervals of the other system is real or not, is undecidable. So reality and observation melt into each other. We might think that this leads to total confusion, but on the contrary, it merely narrows down the true facts. In the old fashioned view, reality starts at the internal distances and absolute time. The different descriptions must leave these and other physical laws intact too. Now with the new relative systems, we will have different internal distances, and times about each other, so seemingly reality shrank drastically. But, we obtain new physical laws, which are absolute in all systems. So the full scope of reality actually increased.
The finer details of how all this can come about starts with an above unmentioned but crucial difference between the two basic old fashioned invariances, distance and time.
Only the inner distances are invariant, while the distances to measure coordinates are obviously not. These are shifted or turned. Travelling coordinate systems mean continuously changing shifts and may involve some initial turns too. Interestingly the usual Descartes coordinate system is not the only possibility. Instead of x , y , z coordinates we can use the d distance from the origin and some angles to tell the

direction of d. In three dimension such angular or polar system didn't become universal for physics, but the simpler two dimensional polar system with (d , α) instead of the Descartes (x , y) did become "physical". This is a very strange story in itself. The discoverer of the polar plane points, was the earlier mentioned third genius Gauss. In fact he acted exactly as Einstein. Instead of emphasizing the simple plane geometrical applications that come out with this new polar meaning, he chased the new physical meaning. And his intuition was correct. There are abstractions of physical reality that use the points of a plane as numbers. These so called complex numbers (by Gauss himself) have several alternative physical applications today. But Gauss died without knowing any of them. That was his punishment for chasing abstraction and caring not for understanding, the true non abstract core of idealism. He gave a multitude of proofs for the Fundamental Theorem of Algebra without revealing the crucial new vision that makes it a plausible topological fact. He could have discovered Topology and through it even sets. So abstraction always knocks on the door by itself if you follow the bigger and simpler truth and don't let yourself be trapped by abstraction alone. Gauss was a big admirer of Newton, so maybe he was hoping to extend his field to physics. He was the head of an observatory and predicted the position of Ceres. That in itself makes him a genius in my book but his own book was probably more demanding.

Lets return to the crucial difference between the invariance of distance and time. While only the inner distances are invariant and the observations use coordinates that are very naturally alterable, time was not only invariant as time interval but as a dimension too. Once we add time to space as the fourth coordinate, then of course it's expectable that the absoluteness is lost. As a double whammy we lose the absoluteness of time intervals too. The new transformations are merely single turnings but in four dimension. This might suggests that a new four dimensional polar coordinate system emerges, but it's simply an application of the complex numbers. Time must be measured imaginarily, that is multiplied by $\sqrt{-1}$. The complex meaning of $\sqrt{-1}$ is quite straightforward because multiplication among complex numbers is turning. So then this mysterious $\sqrt{-1}$ is merely the unit of the y coordinate axis. Indeed, it is 90^o turned from 1, that is the unit of x and repeating this turn, that is squaring $\sqrt{-1}$ we end up at -1. But for the four dimensional turning, this imaginariness loses its polar meaning. The theoretical y coordinate is not the actual y coordinate of space. The only thing that will remain is that the square of time is negative, which means that the crucial Pythagoras theorem becomes $d^2 = x^2 + y^2 + z^2 - t^2$. So turning will mean the invariance of this. This four dimensional meaning was discovered only after Einstein published Special Relativity but became fully embraced by him.

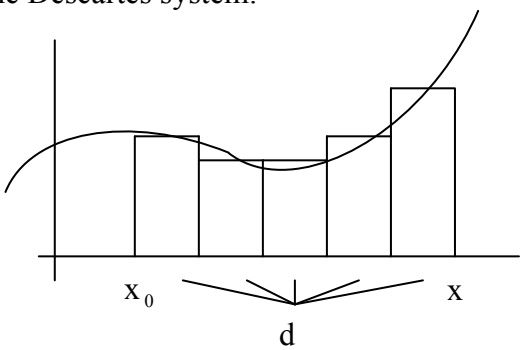
General Relativity might sound like the direct step from translating systems to finding the transformations for systems that move arbitrarily to each other. But strangely, this is merely a side result. In fact we might even doubt that such transformations should come out of new invariances. The old Galilean relativity principle is not true for accelerating or turning systems. When a train speeds up or slows down or turns, the champagne will not pour normally, we all feel strange system forces. Now, we could look for proper system forces that alter the absolute situations of translating systems, but there is one particular external force, that acts very differently. This is gravitation! Just as in Special Relativity the special external reality of light is universally internal to all systems, here now in general, the external force of gravity is internal to all systems. Indeed, all object in a lab will feel the external gravity the same way.

So amazingly an other law of Galileo must be taken more seriously, namely the law of common fall. And again the old Newtonian analytic explanation must be abandoned. Newton said that all objects have a resistance against all forces proportionally to their masses. So a fix force will accelerate less a more massive body. We all know this when we want to move heavy objects. What we don't realize is that the earth gravitation is also just a force! It has to overcome the bodies' resistance too. This reluctance of the bodies to fall down is hidden because seemingly they all obey instantly as we drop them. But if they really obeyed without reluctance then they would move faster if they are heavier since the heaviness or the weight is exactly the force that pulls them down. But they do have a proportional resistance. So the end result is this: The mass makes an object heavy thus it experiences a bigger force of gravity from the earth, but this same mass makes it lazier too to react to the force. The two effects cancel each other and so all bodies accelerate the same. Einstein said again, it's too perfect coincidence so we should use this itself as the start for explaining other things, not get this from others. The new vision is then not that the earth has a gravitational pull depending on the body that we drop, rather the earth bends the space-time which for straight dropping means a radial acceleration in time. The falling body feels no magical force from the earth, rather obeys the curved space-time just like translating bodies keep moving with same speed. If we fall with an elevator or airplane, then indeed we feel nothing or rather we experience artificial weightlessness. In the orbiting space stations we can enjoy this for ever. But this absoluteness of the common fall is only absolute for every point of the space-time, that is same places at same time. Even in an elevator its not perfect, because two bodies a meter apart will not remain so. Indeed, since they fall toward the center of the earth radially, they will get a tiny bit closer to each other as the whole elevator gets closer to the center. So this principle must be applied with total locality only. This is a full circle back to the vision of special relativity:

When I look at my toe, that's not here and now. The light carries the vision to my eyes. What I see is my toe in the past. The only absolute is the light bouncing of my toe, then and there plus the light entering my eye here and now. In between I just estimate what must have happened. Since transformations transform only these absolutely local events, thus the old internal distances and internal time intervals also turn out to be pairs of events, that must be transforming. Nobody denies these transformations to be correct. But some want to have a reality beyond the transformations. And the transformations don't contradict this. Then the question becomes, what deeper experiments can verify those realities. Most extreme is to return to the existence of the absolute ether. But if there is such, then why would it hide itself? And then also, the process of this hiding would have to be that all objects transform when moved through the absolute ether, exactly to keep the measurements indistinguishable. Einstein's view was that just for the sake of easier conceivability, we can't attribute reality to things that are not measurable. So there is an amazingly big coincidence here. He, who didn't care about understanding, only possessing the truth, also created the first theory that defies understanding or narrows it down, to accept measurability. You might even think that I reject his view of this narrow meaning of reality. Well I don't. It's totally logical that there shouldn't be ether and thus also there can't be meaning behind the question, whether the transformations are real or not. But my absolute and irrefutable obedience to this seemingly mind numbing abstraction comes from the real big full circle that Relativity accomplishes.

This full circle is what really motivated Einstein too from the beginning to apply all these revolutionary changes not only in formulas but in basic intuitions. So what is this big "ace" that worth all this? Well I told that he wanted to incorporate light into

mechanics, I even told that he accepted the ingenious electromagnetic equations of Maxwell to be true. Indeed light is electromagnetic wave. But what I didn't tell yet is that Relativity explains magnetism as merely an illusion of electricity! So the magnets that fascinated all of us as kids, the magnets that helped the discoverers of the globe in their compasses, are all just illusions? Yes! But how can systems travelling relative to each other, obeying strange new transformations of space and time lead to the final "elimination" of this medieval mystery? Well that can be seen very easily in its own.
The electrons and protons were the discovered real sources of electricity. But no magical sources of magnetism were found. The iron magnets are merely matters that contain atoms that have electrons orbiting in synchronized directions. These combined and not cancelled motions of electricities that create the real magnets. But much more importantly even the effect of magnetism boils down to merely motion. If a singular electric charge is moving in a magnetic field, then a mystical side force tries to curve its motion. But if magnetism is only the "side forces" of electricities, then literally it is just a side effect of electricity. So then from a moving system we see different composition of electricity and magnetism. The end result, how the electric charges move must be of course invariant. Real charges cause the heat that we enjoy from the sun, but this magical imaginary motion description of their fields is making it possible that the heat can travel faster than any other matter. On the other end here on earth it will influence real charges, conventional matter that builds life. Life that could reveal this real illusion magnetism. To maintain that magnetism is a full blown reality just like electricity would mean that these two realities share each other depending on from where we look at them. Now that would be much worse mystery.
The real present of this truthful endeavour that Einstein accomplished was not only that he solved what he aimed for, the elimination of magnetism, but that we learnt that time is relative too. The deeper problems with this will be dealt in the ten layers of the twin paradox. But now, I turn back to $E = mc^2$.
This flows out with amazing ease from the new transformations of Einstein. That's why Poincare stumbled upon it, just by playing with the transformations mathematically. In fact, quite amazingly, this formula can be obtained conventionally without relativity at all. All we have to do is to assume that if a body is accelerated to different speeds, then its mass can change. The speed of the m mass is denoted as v from the word velocity and then m is actually a function $m(v)$.
The crucial method to establish $m(v)$ is the good old calculus with its two basic concepts, integration and differentiation. Integration is the summing of $f\,dx$ products, where dx means the difference of x values and f is a function of x.
Looking $f(x)$ in the Descartes system:



$d\,f(x_1) + d\,f(x_2) + \ldots + d\,f(x_n) \approx$ area under $f(x)$ because $f(x_1)$, $f(x_2)$, . . . , $f(x_n)$ are the heights of the used rectangles, so we used x_1 , x_2 , . . . , x_n locations in each d interval to get the f value as height.

We could use the minimal or maximal heights of f in every d interval and then we get a totally under or above area by the rectangles. But if d is chosen smaller and smaller, then these two also approach each other and thus, the actual area under f in between them is also approached by any chosen $x_1 , x_2 , . . . , x_n$.
So the hidden condition of integration is that these two approximating minimal and maximal areas must indeed approach each other. Then any rectangle sum is good to approximate the area.
Originally, only nice, smooth f functions were imagined and so the condition was assumed to be obvious. Thus, the main concern was calculation and that's where the name calculus comes from. Later, analyzing the situation showed that f can be ugly, having infinite many jumps or oscillations, so that the minimal and maximal rectangles don't approach each other. That's where the more advanced form of calculus comes as analysis. Just sticking with calculus, the crucial method of integration is then that we neither use the minimal nor the maximal locations, rather the left or right end locations, that is:
$x_1 = x_0 , x_2 = x_0 + d , x_3 = x_0 + 2\,d , . . . , x_n = x_0 + (n-1)\,d = x - d$ or
$x_1 = x_0 + d , x_2 = x_0 + 2\,d , x_3 = x_0 + 3\,d , . . . , x_n = x_0 + n\,d = x$
These two choices are immaterial if d becomes arbitrary small.
Using this method with the first left end choices, we calculate:
$d\,f(x_0) + d\,f(x_0 + d) + . . . + d\,f(x_0 + (n-1)\,d) =$

$$d\,[\,f(x_0) + f(x_0 + d) + . . . + f(x_0 + (n-1)\,d)\,] = d \sum_{0}^{n-1} f(x_0 + k\,d)$$

The $x_0 , x_0 + d , . . . , x_0 + (n-1)\,d$ locations are also called an arithmetical sequence. The sum of it is obviously $n \frac{x_0 + x_0 + (n-1)\,d}{2}$ because this fraction is the average member. But we don't need the sum of this, rather the sum of the distorted f values. Such distorted arithmetical sum can not be calculated by simply n times a half average, but there were old methods for some f functions.
For example, at $f(x) = x^2$ we need:
$x_0^2 + (x_0 + d)^2 + (x_0 + 2\,d)^2 . . . + (x_0 + (n-1)\,d)^2 =$
$x_0^2 + x_0^2 + 2\,x_0 d + d^2 + x_0^2 + 4\,x_0 d + 4\,d^2 + . . . + x_0^2 + 2(n-1)\,x_0 d + (n-1)^2\,d^2$
$= n\,x_0^2 + 2\,x_0 d\,[1 + 2 + . . . + (n-1)\,] + d^2\,[1 + 4 + . . . + (n-1)^2\,]$
$1 + 2 + . . . + (n-1) = n \frac{1 + (n-1)}{2} = \frac{n^2}{2}$.
For the squares, this method would be of course wrong and instead we have to find a formula that can be verified through step by step increment. That is from $n-1$ to n by adding n^2, or from n to $n+1$ by adding $(n+1)^2$. Such formula is:

$$1^2 + 2^2 + . . . + n^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

First of all it is correct for $n = 1$ because the square sum is then 1 and indeed :

$$\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$$

Assuming that the formula is true up to n, the increment from n to $n+1$ is:

$$[\,\frac{(n+1)^3}{3} + \frac{(n+1)^2}{2} + \frac{n+1}{6}\,] - [\,\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\,] =$$

$\frac{3n^2+3n+1}{3} + \frac{2n+1}{2} + \frac{1}{6} = n^2+n+\frac{1}{3}+n+\frac{1}{2}+\frac{1}{6} =$

$n^2+2n+1 = (n+1)^2$ indeed as it should be.

An other way to obtain the formula is to observe that squares can be expressed in a seemingly more complicated way from cubes, but in summation those cubes cancel each other except one remaining:

$k^2 = \frac{k^3-(k-1)^3+3k-1}{3}$ because $(k-1)^3 = (k-1)(k-1)(k-1) =$

$(k^2-2k+1)(k-1) = k^3-3k^2+3k-1$. Then $1^2+2^2+\ldots+n^2 =$

$\frac{1^3-0^3+3-1}{3} + \frac{2^3-1^3+6-1}{3} + \ldots + \frac{n^3-(n-1)^3+3n-1}{3} =$

$\frac{n^3+3(1+2+\ldots+n)-n}{3} = \frac{n^3+3n\frac{n+1}{2}-n}{3} = \frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}$

Using our formula for

$n x_0^2 + 2x_0 d[1+2+\ldots+(n-1)] + d^2[1+4+\ldots+(n-1)^2] =$

$n x_0^2 + 2x_0 d \frac{n^2}{2} + d^2[\frac{(n-1)^3}{3} + \frac{(n-1)^2}{2} + \frac{n-1}{6}]$

As d gets arbitrary small n or $n-1$ both are approximately $\frac{x-x_0}{d}$, so it is:

$\frac{x-x_0}{d} x_0^2 + x_0 d \frac{(x-x_0)^2}{d^2} + d^2[\frac{(x-x_0)^3}{3d^3} + \frac{(x-x_0)^2}{2d^2} + \frac{x-x_0}{6d}] =$

$\frac{(x-x_0)x_0^2}{d} + \frac{(x-x_0)^2 x_0}{d} + \frac{(x-x_0)^3}{3d} + \frac{(x-x_0)^2}{2} + \frac{x-x_0}{6}d$

This sum has to be multiplied by d to get the area and thus, it becomes:

$(x-x_0)x_0^2 + (x-x_0)^2 x_0 + \frac{(x-x_0)^3}{3}$

The last two members were omitted because they have remaining d or d^2 factors and thus, become 0. This final sum can be easily calculated as:

$x x_0^2 - x_0^3 + x^2 x_0 - 2x x_0^2 + x_0^3 + \frac{x^3}{3} - x^2 x_0 + x x_0^2 - \frac{x_0^3}{3} =$

$\frac{x^3}{3} - \frac{x_0^3}{3}$

So the area can be simply obtained by regarding the end values difference in the $\frac{x^3}{3}$ function. This can be expressed by:

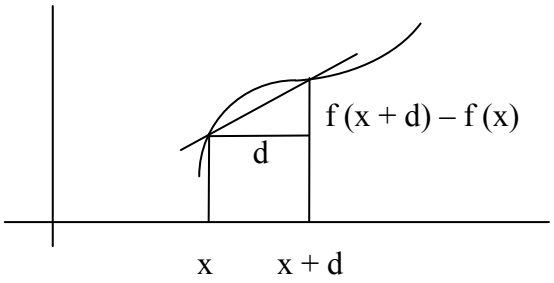
$\int_{x_0}^{x} x^2\,dx = \left[\frac{x^3}{3}\right]_{x_0}^{x} = \frac{x^3}{3} - \frac{x_0^3}{3}$

The $\int$ shows the infinite version of the summation's Σ symbol and [ ] contains the so called "primitive function" of which the infinite sum is merely difference of end values.

The beauty of calculus became that the other infinite approximation for an $f(x)$, the tangent of f at an x value turned out to be the exact reverse of the primitive function.

So, $f(x) = \frac{x^3}{3}$ would have tangent at x exactly as x^2.

Indeed, the tangent can be approximated again with small d difference as the slope of a chord:



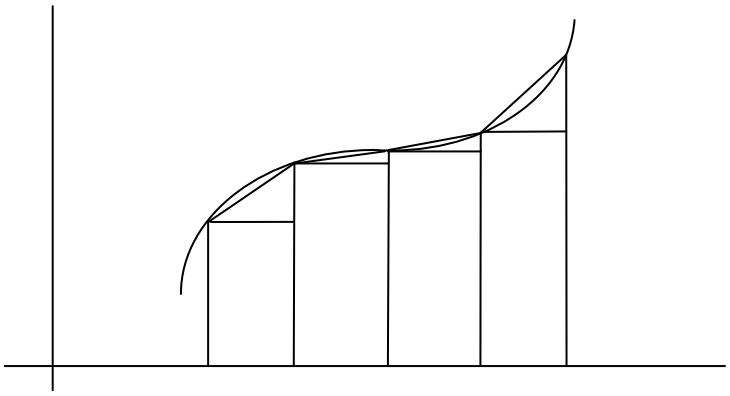
So the slope is $\frac{f(x+d)-f(x)}{d}$, in our case:

$$\frac{\frac{(x+d)^3}{3} - \frac{x^3}{3}}{d} = \frac{x^3 + 3x^2 d + 3d^2 x - x^3}{3d} = x^2 + x d$$

which becomes x^2 if d goes to 0.

Here the function obtained as the result of differentiation is called the "derivative" and is denoted as $f'(x)$.

This reversal of integration and differentiation or primitive functions and derivatives is not a magical coincidence and can be seen directly, universally too:



The integration of the derivative is summation of the d times tangent values, which are actually the little heights of $f(x)$ values in the triangles. So, their sums become the total change of the $f(x)$ function from x_0 to x. Formally:

$$\int_{x_0}^{x} f'(x)\,dx = \left[f(x) \right]_{x_0}^{x} = f(x) - f(x_0)$$

The only annoying little nuisance is the $-f(x_0)$ fix value. This is natural though.

The derivative can not depend on f fully, only on how it changes.

Shifting an $f(x)$ function up or down, we get the same tangents at every x.

When Newton developed these ideas, he had physical meanings behind both integration and differentiation. The crucial applications though are not the reverse.

For integration, it is the concept of work, namely as integrating force in space, while for differentiation, it is the speed as the differentiation of space itself by time.

Both of these involve space, which is three dimensional, so calculus was used for the three coordinates or for vectors. For our arguments, we'll simply use an m mass moving along the x axis, so we won't need vectors. As I said, the two forms are not directly reversed, that is work and speed are not opposite of each other, but there is a

connection through the force itself. That was the big physical discovery of Newton, that force is related to speed, namely, to the change of speed, that is to its derivative. This is what we call today, acceleration. But this includes slowing down or turning. Before Newton, it was not emphasized that acceleration is crucially different from just motion in general. It was observed that fix speed motions, like merely sliding with a boat can keep going very long without any force. But they didn't realize that this very long is actually infinite. The boat only stops because the water is not perfectly smooth, it causes friction, that is force against the gliding. Similarly, any ball rolling on a table should keep on forever. Truly forceless motions keep on forever. These are the translations we already used and if forces are present, then their total will determine the acceleration. But as we also explained, the mass of the body counts too. Bigger mass is harder to accelerate. So we have the famous $F = m\,a$ formula. The force is proportional to the acceleration and the mass too. But, this proportionality was meant for regarding different masses. For the motion of one fix m, the acceleration is only proportional to a. This a is of course the derivative of the speed $a(t) = v'(t)$.

In spite of this, it was already clear to Newton that the m mass is already useful to be multiplied with the speed. In short, mv has a meaning. It can be called the momentum of a body. This is what causes the push or kick it can give on impacts to other bodies. In fact, it is also what changes when a push or kick is received from other bodies. So in short, the exchanges of part of these momentums are the impulses, the interactions. The differences in momentums or the impulses cancel each other or to put it another way, the total momentum of all interacting bodies remains the same.

This all came out from the $F = m\,a$ law because Newton also introduced the action reaction law that claimed opposite but equal forces among interacting bodies.

But for one single body, the meaning of $m\,v$ were not really used. Of course, $F = m\,a$ could also be written as $F(t) = (m\,v)'(t)$ because m being constant can be taken out of the derivation, so $F(t) = (m\,v)'(t) = m\,v'(t) = m\,a$. Is this a useless overcomplication? Not really. We could imagine a rubber ball into which we pump water through a tube. Then using an F force to accelerate this, while we change its mass too, we would have the dilemma, how big is the force? The logical is exactly $F(t) = (m\,v)'(t)$. In spite of this simple situation, it was never followed through, what it would really mean if m could change just by itself. Namely, we could continue the examination from the force to the work, that is done on the body to accelerate it. And then, we could drop in the crucial assumption that this work melts into a total energy that the body possesses. So then, if $m(v)$ denotes the changing mass with speed, $p\,m(v)$ would be its total energy, where this p is merely a universal proportionality constant. This assumption that the full energy is proportional to the mass is very natural. We already saw that resistance against force and gravitation are also proportional with the mass. The $p\,m(v) - p\,m(0)$ change of this total energy, when accelerated from 0 to v, would have to be the work invested and calculated as the integral of force with the distance, that is $\int_{x_0}^{x} F(x)\,dx$. Here x_0 is the place where we started to accelerate and x is where the body reached its v speed. So:

$$E(v) - E(v_0) = p\,m(v) - p\,m(0) = \int_{x_0}^{x} F(x)\,dx = \int_{x_0}^{x} (m\,v)'(t)\,dx = \int_{x_0}^{x} \frac{d(m\,v)}{d(t)}\,dx$$

$$= \int_{0}^{v} \frac{d(m\,v)}{d\,v}\,\frac{d\,x}{d\,t}\,d\,v = \int_{0}^{v} (m\,v)'(v)\;x'(t)\;d\,v = \int_{0}^{v} [m'(v)\,v + v'(v)\,m(v)]\,v\,d\,v$$

$$= \int_0^v [m'(v)\, v + m(v)]\, v\, dv = \int_0^v [m'(v)\, v^2 + m(v)\, v]\, dv$$

We applied changes in the variables and their differences. Then used the simple rule that $(f g)' = f' g + g' f$, plus that $x'(t) = v$ and $v'(v) = 1$.
Now we can reverse the integration to derivation and so:

$[p\, m(v) - p\, m(0)]' = p\, m'(v) = m'(v)\, v^2 + m(v)\, v \;/ - m'(v)\, v^2 \;, :(p - v^2)$

$m'(v) = \frac{v}{p - v^2}\, m(v)$. The solution of this is $m(v) = q\,(p - v^2)^{-\frac{1}{2}}$.

Thus, a new constant q beside the already used p is allowed.
The verification of this solution is easy if we apply the universal law of derivatives for exponents: $(x^e)' = e\, x^{e-1}$ plus that if a whole $f(x)$ is used in place of x, then we must multiply with $f'(x)$ too. Indeed then, $m'(v) = [q\,(p - v^2)^{-\frac{1}{2}}]'(v) =$

$$-\frac{1}{2}\, q\,(p - v^2)^{-1\frac{1}{2}}\,(-2\, v) = v\,(p - v^2)^{-1}\, q\,(p - v^2)^{-\frac{1}{2}} = \frac{v}{p - v^2}\, m(v).$$

Now q can be established from the $m(0) = m_0$ resting mass:

$$m(0) = q\,(p - 0)^{-\frac{1}{2}} = q\,\frac{1}{\sqrt{p}} = m_0 \text{ so } q = m_0 \sqrt{p}.$$

Thus, the mass function is $m(v) = m_0 \sqrt{p}\,(p - v^2)^{-\frac{1}{2}} = \frac{m_0}{\sqrt{1 - \frac{v^2}{p}}}$.

To establish p, we have to make a new assumption. Namely, as we see,
$\sqrt{1 - \frac{v^2}{p}}$ can become 0 if $v^2 = p$. This then means $m(v) = \infty$.
But at what speed should this meaninglessness occur? Well, the light is the fastest thing we know, plus it seemed controversially, always the same. So this is a perfect choice as the universal top speed. Thus, $p = c^2$. Then the final formula for the changing mass is: $m(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_0}{r}$, with r as the relativistic factor at v.

This r is always a factor between 0 and 1, becoming 0 only at light speed and at normal low speeds almost 1.
Now we can obtain the famous formula for the total energy E.
$E(v) = p\, m(v) = c^2\, m(v)$. In short, $E = m\, c^2$. At $v = 0$ speed $E_0 = m_0\, c^2$.

So a mass, even at rest contains $m\, c^2$ energy. This comes from its internal atomic and sub atomic motions, that we were able to obtain without even knowing what's going on inside. Now that's real magic!

The c is a huge value and c^2 is even bigger, so a cup of water contains an incredible total energy. Atom and even hydrogen bombs are still only able to tap into a fraction of this total energy. This means that after the explosions, most of the mass of the bomb itself is still around as normal resting mass, but of course, can not be seen in the destruction that we caused. This can easily give the old fashioned view, that the fraction of the resting mass that "disappeared" was converted into the huge blast as "work". But if we think a bit further, then we can realize that every body that was moved by the blast, gained speed, so increased its mass exactly through our new $m(v)$ vision. Of course, these motions came to rest, so they probably re-gained their rest mass, but then again, they influenced other bodies. The point is that even the tiny little mass that caused the blast didn't disappear in the universe. So, the mass did not change into or converted into energy, rather exposed itself from hidden rest energy to actual work. The total mass of the universe, just as the total energy of it, didn't change. So mass and energy are the same, indeed, just two faces of a coin as the douchebag professor from New York university said. What made him a douchebag is what he didn't say. And the crucial thing is not clearly said above either.

Converting mass into energy or keeping them both fix, still can be misleading.

The oldest idea was the conversion. When we burn a campfire, the wood seems to disappear, so heat is created from matter. Then when they were able to analyze matter, and measure it more exactly too, it turned out that the wood doesn't disappear at all. The carbon combines with oxygen from the air and the formed heavier CO_2 carbon dioxide molecules are still light enough to be carried away, one by one, into the air. So the whole burnt wood (except the ash) is still there in the air. But then where does the heat come from? Well, they started to think that energy is merely a side effect of changing matter. Actually, already at the campfire, there is a strange duality of the heat that predicts something wrong about this assumption. Namely, only some of the heat is lead out into the air. Some radiates. We could even use a thick glass wall next to the campfire and still feel heat behind it, in the cold air. For the sunlight, this is even more obvious, practically as the green houses, that keep warm even in the winter, but also theoretically if we realize that all the sun's heat must come in this radiating form, because there is no air between the sun and the earth. Only when sunlight enters the atmosphere do we have the duality of heats as hidden air motion and radiation still present. On the moon, only radiation is present, but the heats and lights are not totally sharp there either, because of reflections from the surfaces.

Instead of a campfire, we can regard a car that exhausts into the air, the burnt form of the fuel. The mass of the petrol is again the same as the exhausted fumes. So where does the work of the car come from? Well, the new measurements of science were not accurate enough to show that we were wrong! The wood or petrol doesn't burn completely into fumes. An unmeasureably tiny portion is used to actually get the heat or drive us around. These are even less efficient energy sources than the nuclear bombs, so really tiny portions of matter seemed to convert into energy. But of course, again, our wisdom not only tells about this tiny portion, but also about that it didn't convert at all. The heat of the campfire or the work of the car effect other matters, increasing the mass of the surroundings exactly with that tiny portion that was used for the temporary energy form. Even if we assume that the heat is not encountering any objects and merely leaves as radiating heat, this radiation itself has mass too. It has resistance against force and even gravitation too. So mass never converts, it always remains continually but maybe not as easily visibly.

The motions that use force can be calculated as works. These increase the m_0 rest mass to bigger $m(v)$ resistance or gravitation, that is fully functional mass. But this $m(v)$ is relative to a system in which we measure v. These works were not only used

in our tricky derivation of the $E_0 = m_0 c^2$ real or rest energy, but they are indeed, the methods of how energies transfer too. So the tiny little portion of the petrol mass that turns into the driving in the city is going back to the city as the resistance against our motions. The air, the road, and especially the used break pads that "turn" into heat again. So we can't deny reality from the $E(v)$ non absolute energies or $m(v)$ changing masses. The conservation of mass and energy of course claims only the absolute total of all E_0 and m_0 of the universe. The changing relative $E(v)$, $m(v)$ values of the bodies are observational "subjective" measures of how the "absolute" E_0, m_0 values of the bodies change, that is distribute between them in the universe. So as we see, even the absolute is not really absolute, only in total, and thus, why should we abandon the subjective pictures of the distributions.

A formalist trend entered even physics, exactly against the $E = m c^2$ formula used in its wider, that is $E(v)$ and $m(v)$ meaning. These "thought officers" claim that the non absolute E, m are confusing or undidactical. This is a sheer nonsense, because nothing is more clear than the fact that if we travel with a high speed relative to a mass, then it will have different observational speeds relative to us. So, it is obvious from the start that all our deductions from such speeds are relative. Yet, we made a connection between the work and such relative mass. Should we throw out all such relative contemplations? Exactly such relative contemplations can lead to the recognition of absolute concepts like the invariance of the four dimensional space time length. In fact, as it turned out, all other physical quantities form such four dimensional invariant vectors. And that's what these new Formalists want to enforce.

There is nothing wrong with emphasizing the four dimensional picture. In fact, we can formalize it even better if we use $i = \sqrt{-1}$ from the word imaginary, and convert time to space like units by multiplying it with c, the speed of light.

So in short, the $(x, y, z, i c t)$ is the space time point or vector or event.

Such four dimensional vectors are indeed, the skeleton of Relativity. But we can still put flesh on it. Only then will it become flesh and blood, real usable intuitions.

The most important absolute conquest of Relativity, the solution of magnetism is also a flesh and blood application of the skeleton. In fact, if we use this analogy, then we can even say that the initial $(x, y, z, i c t)$ point is not even the most basic four vector. Or if this $(x, y, z, i c t)$ is the skull or the origin, then there is a spine, which grows out of this, but is more fundamental in supporting all the others. Also in a sense, this spine is even simpler than the four dimensional space time point or event. How can that be? What can be simpler than space and time? Well, just think about what the true assumption of Relativity was. The absoluteness of light speed! As I said, that light speed should not be imagined as a motion of point, it is more elementary. Well, speed in general can have such elementary importance. But to grasp that crucial elementary speed concept, first we have to identify something that was not mentioned yet. We said that $\sqrt{x^2 + y^2 + z^2 + (i c t)^2}$ is the new Pythagoras distance, which is fix, that's why we have merely a turning as transformation. But what is this four dimensional length itself? Quite amazingly, it is itself an imaginary τ time. So:

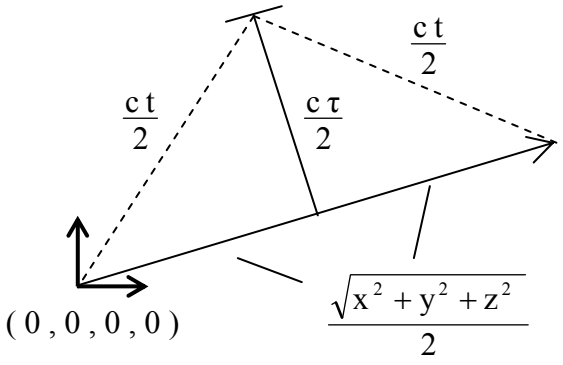
$i c \tau = \sqrt{x^2 + y^2 + z^2 + (i c t)^2}$. This τ is the time of a clock that travels from the origin $(0, 0, 0, 0)$ to $(x, y, z, i c t)$ with fix speed. This of course is a contradiction, because if (x, y, z) is too far from the origin, then only faster than light motion could reach there by the t time. Yet, our formula allows this with its use of i in time as coordinate. If τ is real, then $i c \tau$ is imaginary, so the event has imaginary length and so $i c \tau = \sqrt{-1}\, c \tau = \sqrt{x^2 + y^2 + z^2 - (c t)^2}$ so squaring:

$-c^2\tau^2 = x^2+y^2+z^2-c^2t^2$ or $\tau^2 = t^2-\frac{x^2+y^2+z^2}{c^2}$ or $\tau = \sqrt{t^2-\frac{x^2+y^2+z^2}{c^2}}$

So the imaginary four dimensional length gives the very real $\tau > 0$ time.

Indeed, this means $t^2 > \frac{x^2+y^2+z^2}{c^2}$ or $\sqrt{x^2+y^2+z^2} < c\,t$,

that is the place is closer to the origin than the light would go under t time.
If (x, y, z) is further than this, then in reverse, only imaginary τ value can be.
This is a pure abstraction but we still accept the invariance of the imaginary clock times, even for these unreachable events too.
The derivation for the positive τ values goes by a direct heuristic argument of Einstein, that he used originally, not for this invariant τ, rather for the transformations themselves. Lets use a mirror that reflects a light starting from $(0, 0, 0, 0)$ to exactly $(x, y, z, i\,c\,t)$:



From the clock's view, the light merely went back and fourth on the $\frac{c\tau}{2}$ distance.
Now, assuming that perpendicular distances to the relative speeds of the systems are not differing, then $\frac{c\tau}{2}$ is usable in the $(x, y, z, i\,c\,t)$ system too, and so by Pythagoras theorem:

$\left(\frac{c\tau}{2}\right)^2 = \left(\frac{ct}{2}\right)^2 - \left(\frac{\sqrt{x^2+y^2+z^2}}{2}\right)^2$ which is exactly $\tau = \sqrt{t^2-\frac{x^2+y^2+z^2}{c^2}}$.

Amazingly, if we calculate the v speed of the clock in the $(x, y, z, i\,c\,t)$ system, it is $v = \frac{\sqrt{x^2+y^2+z^2}}{t}$ from which $x^2+y^2+z^2 = v^2t^2$ and with this:

$\tau = \sqrt{t^2-\frac{x^2+y^2+z^2}{c^2}} = \sqrt{t^2-\frac{v^2t^2}{c^2}} = t\sqrt{1-\frac{v^2}{c^2}} = t\,r$.

So the same r relativistic effect decreases t to τ, as increased m_0 to m by classical calculus. But that's not all!
Now we inject the abstraction of τ back into the four dimensional event in an ingenious way that will bring about the m increment too. First, lets find the promised four vector that is simpler than the event. As we said, this coincides with the universal speed concept. Instead of the usual velocity vector which would be:

$V = (v_1, v_2, v_3) = (x'(t), y'(t), z'(t)) = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$, we'll use:

$U = (u_1, u_2, u_3, u_4)$ four dimensional vector. The U can stand for universal.

The differentiation or derivation by t is clearly not absolute, since it is used in the $(x , y , z , i c t)$ system. So lets use τ for differentiation, that is:

$u_1 = \frac{d x}{d \tau}$, $u_2 = \frac{d y}{d \tau}$, $u_3 = \frac{d z}{d \tau}$, $u_4 = \frac{d (i c t)}{d \tau}$. With $\tau = t r$, these are:

$u_1 = \frac{v_1}{r}$, $u_2 = \frac{v_2}{r}$, $u_3 = \frac{v_3}{r}$, $u_4 = \frac{i c}{r}$

So the first three coordinates are merely the relativistically altered speed coordinates, but a totally new fourth coordinate appeared too. So, in another way, $U = (\frac{V}{r} , \frac{i c}{r})$.

What is the length of this U ? Since the length of the $(x , y , z , i c t)$ four dimensional point is $i c \tau$, thus the derivative by τ will have $1 – 1 = 0$ exponent, so τ disappears and it is merely the $i c$ constant. So, U indeed has a simpler invariance than the space time, because the invariance reduces to the constancy of the speed of light.

Constancy among numbers means that there is no variable or it has 0 power and thus, by the derivative rules of exponentiation, the derivative is 0 too. But U is a vector. Constancy of its length, visually means that the end is on a fix sphere. Of course, in four dimension, this can't be seen. Still, then the derivative of U is not 0, only perpendicular to U. The tangent as derivative "means" actual tangent to the sphere.

This perpendicularness of U to its derivatives can have different meanings according to what variables we use for derivation. Differentiating by the t time, that is, forming $\frac{d U}{d t}$, we clearly won't get an invariant four vector again. But differentiating by τ, we do. This $\frac{d U}{d \tau}$ is the four dimensional acceleration and thus, it is always perpendicular to the U speed. In spite of this better direction, we still pursue the other $\frac{d U}{d t}$ non invariant vector for meanings. Though it is not invariant by the transformations, it is still perpendicular to U. This perpendicularity also means that $\frac{d U}{d t}$ is perpendicular to small $d P$ differences. If P is the $(x , y , z , i c t)$ event.

Indeed, $d P$ is same directional as U. The change rate of U in a system's t time is some kind of system acceleration, but not quite correctly, because U was already using $d \tau$. In a better way, if we multiply U with the m_0 rest mass, then we definitely get a four vector with invariant $i c m_0$ length. This $m_0 U$ vector should be the momentum four vector. Then it's also expectable that $\frac{d m_0 U}{d t} = m_0 \frac{d U}{d t}$ is the four dimensional force. This can't be invariant again, because we use $d t$, but should give some clues about the fourth coordinate of U and $m_0 U$.

The crucial extra concept we need is the multiplication of vectors. This simply means multiplying the coordinates and adding them together. Thus, we obtain a number from the two vectors. But the meaning of it as a product is that, it is actually the product of one with the projection of the other. So perpendicularity exactly means 0 product.

Then $\frac{d U}{d t}$ being perpendicular to U or $d P$ means $\frac{d U}{d t} d P = 0$. This still means nothing yet. But with the $F = m_0 \frac{d U}{d t}$ force meaning, it means $F d P = 0$ too.

But force times little distance, means a little work. At least, in classical sense, that is in the first three coordinates of the product. So:

$$F\,d\,P = m_0 \frac{d\,U}{d\,t} d\,P = d\,W + m_0 \frac{d\,u_4}{d\,t} d\,(i\,c\,t) = d\,W + m_0 \frac{d\,\frac{i\,c}{r}}{d\,t} d\,(i\,c\,t) =$$

$$d\,W + \frac{m_0\,i^2\,c^2}{r} = d\,W - \frac{m_0\,c^2}{r} = 0\,.\ \text{So,}\ d\,W = \frac{m_0\,c^2}{r}$$

Thus, $\frac{m_0\,c^2}{r}$ is changing exactly as the total energy, so $E = \frac{m_0\,c^2}{r}$ + constant.

Assuming that this constant is 0 because our relativistic arguments are lucky, we get a meaning for $m_0\,u_4 = \frac{m_0\,i\,c}{r}$ as $\frac{i\,E}{c}$.

So the fourth coordinate of the $m_0 U$ momentum vector is the $\frac{i\,E}{c}$ total energy.

The invariant length of it is $m_0\,i\,c$, the rest mass.

So it's true what the new fascist thought control agents say. E is just a component of the $m_0 U$ vector. Its length is fix and means the m_0 rest mass. But, E has resistance to force and gravitation, so behaves as we recognize mass. Thus, there is nothing wrong about denoting it with a proportional m too, according to $E = m\,c^2$.

In reverse too, the m_0 rest mass can be envisioned as hiding the $E_0 = m_0\,c^2$ energy.

There are three bad things about this new formalist movement.

One is that they infiltrate the education system. So while in popularized science, there are plenty of opponents and sane voices, the tertiary teachers want to appear professional by obeying the hard line reduction to abstractions.

Secondly, this reduction to abstractions is fought in the name of clarity. Well, clarity is emptiness without intuitive chunk, without the flesh on the abstract skeleton.

Thirdly, they are using name dropping by quoting Einstein on this subject. First of all, Einstein didn't give a rat's ass about people and understanding. Secondly, he himself started with very different and varied concepts about mass and energy. As I said, he embraced the four dimensional picture and in a sense, went overboard later, by saying that this should be the guideline.

The dragging of Einstein into this argument is in the same vain as the dragging of Gödel into the new official trend to rename all effectivities into computabilities. Here, instead of emphasizing the real surprise, that it's not the decidability, rather the derivability is the fundamental concept, so we could use effective for this, they keep the old mistake of the recursive approach that obtained derivability from decidability with the artificial method of partial functions and the μ operator. Then they simply renamed recursive to calculable. The point is that effectivity is not finished, just because we accept the Church thesis. Gödel felt all this unfinishedness and finally merely expressed an acceptance of the Turing machines to be the purest abstract form of effectivity. The emergence of computers does not justify the computable naming, in fact forbids it. Because there is no intuitive direct connection. Only to the computer scientists is it "plausible" that there are universal machines or what this means at all. For normal people, a computer is an appliance to go on the net. So here too, not understanding, rather abstraction is pursued. By the way, just remember that Relativity is not finished either. That's why Einstein wasted his last decades. These two geniuses, Einstein and Gödel were trophies of Princeton, just to elevate the status of the school. Princeton gave them free money, so they could walk in the park and criticize America in German. Instead of changing America, by teaching.

So to finally recap the three meanings of $E = m\,c^2$:

The first is merely the amazingly big c^2 factor, which explains why a false mass conservation was observed when things burn into waste, while "creating" energy. In these burnings of wood, petrol or even nuclear matters, the created work or blast steals some mass, but only $\frac{E}{c^2}$, so small, that seemingly, the waste is the same as the original mass.

The second meaning is that even this is not lost though as mass either, so there is no conversion of mass into energy either. Indeed, those works or blasts increase the masses in the surrounding, or even if they just travel as radiation, they still have tiny mass.

Thirdly, the view of this mass increase process can also be obtained by $E = m\ c^2$ for the bodies that gain the heat. Its parts will move faster. So it's not that the heat turns into mass directly, rather this speed change of the particles in an m_0 will increase m_0 itself. This third meaning is a closed application of the wider $E = m\ c^2$ relativistic or system dependant meaning. By using for all parts of a body, in a central system, the system dependence disappears, the invariant m_0 is changing. And indeed, invariance doesn't mean being absolute for a body, if it interacts with others. Only the total is absolute. The changes between bodies should not be visualized according to the thought police. But they can be, by non invariant concepts.

What the whole earlier mentioned formalist abstraction fetish hides, is a deeper puzzle about physics in general:

The truth is that strictly speaking, in physics, all we have is "correct" equations. Of course, "correct" needed a total rewriting as classical physics changed into Relativity. It was also expected that a third rewriting must come, that would involve Quantum Mechanics. This didn't happen, which turns the puzzle into a pending mystery. But first, lets see the puzzle. In fact, first lets observe that mathematics went through a "rewriting" too. But this amazing new math has absolutely no place in physics yet. To put it even more bluntly, to any real mathematician, physics is a joke with its "sophisticated" equations. It is strange that in one of the rare occasions, when Einstein expressed his views, he pin pointed as the biggest mystery of the universe, the fact that abstract human mathematical equations rule matter, even beyond the particular individuality of the universe. This healthy idealism of course, totally ignored the deeper mystery of understanding. This is the real "beyond" of human thought, and the equations flow from this. So, the coincidence is one level deeper. But it also ignored the problem of math being one step ahead already. Formally, it is just a parallel renewal to Relativity, as opposed to old mechanics. In fact, new math has the same formalist traps of abstractions, accepted blindly. But why is the parallelity hollow? Why is new physics, Relativity and Quantum Mechanics using old math? And how come Einstein didn't care about this? His best buddy at Princeton, Gödel, could have enlightened him. But their relationship was as formal as their truth searching. They both wanted to solve the puzzles of their fields, while ignoring the much bigger common puzzle of understanding. The unrelatedness of their two pursued fields, was a coincidental reminder of the tragedy that still goes on. We don't know shit and we are even afraid to talk about it. Those mathematicians that feel physics to be a joke, are correct, but they fail to recognize the darkness behind this joke. The logic of math doesn't apply to physics. Derivations are not the essence of physics. But the final formulas are neither. Just to recognize the non derivationalness is a healthy step though. The way physics text books try to "develop" the equations is a real joke. Of course, they can't just state the formulas and then say, "That's it folks.". The experiments confirm these totally abstract frames, that were discovered by some

exceptional super-humans, but you don't have to worry about how. Just accept it. Obey the law and consume. Oh, sorry, that's the rule of society. It was very refreshing to read Richard Feynman's manifesto against the derivational mania in physics. This mania involves the over exactness of math requirements too. That was very personal to me. My older brother, Peter, was kicked out of the electrical engineering faculty, because he failed the insane abstract math requirements. He went to a college instead and still became a very good engineer. Earlier, he was a whiz kid builder, making radios and electric organs and so on, in high school. My math faculty had Akos Csaszar as professor, though luckily he didn't teach my year. So I could encounter the true teaching genius of an unknown lecturer Laszlo Czach. Csaszar's wife, who taught at the electrical engineering, wrote the text books for my brother. He threw them out while I was still in high school. At that time, I enjoyed them and only later realized how putrid they were. Indeed, it took me even a longer time to see that the whole math education separated from physics, is a conspiracy. For Newton, it all made sense. Calculus was born out of physics. Now, even in high school, they follow two separate lines. A useless abstract math and a hollow physics with empty formulas borrowed from math. They avoid vectors in math, to make it more precise or easier to formalize and then physics has no real math behind it. Devilishly stupid, so that even the math and physics teachers can converse about nothing in the staff room. But lets return to real physics, to the final of our subjects, the Twin Paradox.

This is about the relativity of time intervals. The expression "time interval" might sound as a mere pretentiousness, that indeed we hear on TV. Politicians say things like, "At this particular point in time, we can't tell more about this." or "In this time interval, the growth was undeniable.". Here, in Relativity, the word interval has a meaning, namely because only four dimensional events or points are to be transformed. So the aging of someone has no direct meaning if he is moving around. If he stays at one fix x , y , z location, then his time coordinates change only, and then the two $(x , y , z , i c t_1)$ and $(x , y, z , i c t_2)$ events, determine a four dimensional vector that could be regarded as the time interval. The reason for this is an idea that is again one of those historical, flesh and blood ones that makes sense out of the skeleton, strictly four dimensional transformations. Einstein emphasized that the absoluteness of light speed, not only makes a restriction on how relatively moving systems should view the universe, but also offers a very plausible method of synchronizing the clocks or times of a fix system. If there is a master clock at the origin, then all places can know the exact time at every moment. Indeed, the origin can send out signals, telling the time, continually. A place will obviously get this signal with a delay. But knowing its own location, every place can deduct the delay. But why should the signals be light speed ones? Well, they don't have to be. Any fix speed can be chosen. Only when we look at how other moving systems would view the synchronized times as unsynchronized, in its own system, is it convenient to use light speed, because that is the same for all systems. In the abstract treatment, all these ideas disappear. Then, the invariance of the four dimensional length is all that matters. This means a turning. But what turning exactly? That still has to be determined from the relative speed of the systems. So it relies on this old fashioned concept. In fact, we also assume that if one of the systems claims V relative speed, then the other claims $-V$. Unless we also assume an initial turn of the coordinates. With that we have to assume a different W as the opposite of V and this initial turn will beautifully melt into the final four dimensional turn. But there is a crucial, additional assumption too. Namely, that the transformation is linear, that is fix proportional from all coordinate values. Every coordinate of one system is a fix linear combination of the others, that is $m_1 x + m_2 y + m_3 z + m_4 t$. Of course, we have four of these m_1 , m_2 , m_3 , m_4

multipliers, so actually, we have a four by four matrix of numbers, that tell exactly the transformation. The invariance of the length, that is the fact of having a turning and the additional informations of the V, W relative speeds, then determine this matrix. We have to admit that this abstract view has some advantages against the older synchronization and distortions to regard the transformations.
In fact, we have more insight into some old facts too:
Linearity is exactly the old Euclidean principle of space. The parallelity axiom guarantees the conservation of angles when pictures are blown up or down, proportionally by distances. A picture of a geometric argument on a black board or in the sand, means its truth in galactic or atomic sizes too. I accept Kant's crucial discovery about the a priori-ness of space and time. It's an experimental fact that people possess these intuitions that are not learnt. So then what about the new abstractions of the four dimensional linearity. Is this a generalization of the three dimensional or it contradicts that? Neither and both! In one system, the synchronizations are exactly the use of the Euclidean intuitions. Infinite, proportional space. But then, when we regard different moving systems, a bigger picture emerges. These infinite, proportional spaces are illusions or appearances. They are still usable, but won't give answers to all questions. Now, an old proportional Euclidean space suggests a space with infinite many stars, which is incorrect for our universe. The alternative, finite many starts moving away from each other, placed in an Euclidean system looks ridiculous. Infinite empty space, with matter merely trying to grow from a single explosion to a cold, but still finite dust spot in the black, infinite nothingness. Is that single explosion the absolute center of that space? Well it would be, but that space is not absolute. So there is no center. The more we go towards the multitude of equivalent systems, the more we get a meaningful acceptance of the finite centerless universe we live in.
Yet, to hop on the band wagon, deny Euclidean space, idealism, Kant, and obey the new idols of abstractions, is a cowardly and evil act. Indeed, it is the weakness of trying to be something big, that is alien to one's actual actions and thoughts. A denial of common sense for the sake of being exceptional. And it's evil because it goes against others. Indeed, these abstraction worshipping Formalists don't even want to "elevate" others from their old common senses to the new abstractions, merely enforce it. This of course, works better, because this way, those who obey will become enforcers themselves. That's how Formalism spreads.
Society created this environment of isolation and suppression of understandings. Society wants to impose itself on human thought. Scientific thinking is only part of this. The main goal is a thoughtless race, an ant farm. The fear of being expelled, not to belong is already a deep motivating force. Work was a necessity of survival. Now, unproductive work became the justification of existence with family as its micro unit, the emotional glass house and consumption as its reward. Freedom is understanding, and this is what free society wants least. The important thing is not how many break the law, rather how many are afraid to break the law. The fear today is deeper than the cavemen's fear was of killer predators and an unconceivable world. But just as then, only the individual caveman counted, who drew pictures, lit fire. Today too, only the individuals can keep understanding alive. This is still the early history of a thinking race in fight against itself. This earliness of our history is appropriate when we really turn finally to the Twin Paradox of time.
The time interval of a point, that is the aging of a person transforms exactly as the $\tau = t\,r$ clock time we used for the invariant length. That is, $t = \overline{t}\,r$ where $\overline{t}$ is the time observed in a system that moves with v speed relative to the one where t was spent still. Indeed, by the explained synchronizations, we might as well imagine t

spent in the origin, so then for the moving system, t is exactly the τ clock time and $\bar{t}$ is the old t. The big difference is now, that we look in reverse too. At that time, we didn't care what the clock would see. But what would? Simply an other clock, moving away with opposite speed. Indeed, we can imagine clocks in every system. When the systems start moving, they are at the same place, so the clocks can even be set to 0. Of course, this idea of start is incorrect, because the speeds were assumed to be fix forever. So, merely a coinciding could happen of the origins and we start, or rather, 0 time from there or to that. Even this coinciding of the origins is not a big deal, because with the synchronizations, we can also relocate origins, so any point of the systems can be used as origin. Since r contains v^2, thus V or $-V$ give same r factor, so both systems deduce that the other's moving clock time is r times of his standing time at every place. These places of course, have synchronized times to its origin, so the standing system, simply can talk about his time. In short, both systems claim that a moving clock slows down by r factor. Now obviously, twins can't just meet in space at a single time for setting their clocks to 0. They had to be born together. So we assume this. Even that they lived together until one of them leaves for the cosmos. After a short acceleration of his rocket, he will spend a few years traveling with a fix high speed. Then he turns back, or rather slows down and accelerates backwards shortly and then comes back to earth with same fix speed. Finally as optional he can either stop and return to earth to live or just wave to his brother while passing by earth. In either case they meet again properly or only for a moment, so an interesting dilemma is whether their opinions about the slowing down of clocks in motion will apply to human lives. So how old they seem to each other?

This is how usually the paradox is introduced, but the real root of the whole intrigue in the question is much deeper. Namely an instant intuition that cuts through our abstractions. The abstract picture is simple. The transformations are linear so they will be linear for distances and time intervals too. Usually the symmetricalness is emphasized, that is the fact that both systems claim same distortions of distances and time intervals viewed in the other. But there is a much more elementary distinction between distances or time intervals. It's both obvious that ten meter decreases ten times as much as one and ten hours decreases ten times as much as one. But while distances have this potential bigger smaller takings, time intervals accumulate because time only goes forward. This is something that we still carry as baggage from our old views and no matter how we pretend to be hip and abstract, it reveals itself in how we feel. Looking at the world in motion, we can imagine a distorted world. A space for sure, that like rubber compresses or stretches. But the distortions of clocks is much less obvious as merely a bit slower tick rates. We wouldn't even hear much of it in a bypassing world. Instead we know that then by linearity these distortions accumulate too. The clocks not only tick slower, they carry all the slowdowns since they were synchronized if they were ever. So a lot of hidden assumptions are jammed in here.

So the first level of the Twin Paradox is that it hides the time accumulation intuition. In other words the first level is the recognition that all the levels will be actually about the accumulation distortion.

The second level is a certain escape from the whole big problem by saying that a return in itself is not really that different than having opposing views at different places. Why couldn't both of them experience what they predict by science. The clocks slow down, times accumulate, so both brothers must appear younger. But here we definitely have to distinguish the mentioned option of stopping and hugging or just passing by and waving hello. Being in motion to each other can easily cover such symmetrical delusion, but stopping and living together with a continuous delusion of

the other being much younger seems really contradictory. So this second level is actually a conflict of two levels. Local confrontation or system confrontation.
The third level is merely the truth. Neither level of conflict is possible. After return, whether only for a second or forever, only one of them can be right. They cant have conflicting ages for each other, or rather there has to be a truthful age relationship.
The forth level is again a new conflict that arises from the recognition that they weren't completely symmetrical systems because the leaving brother had the short accelerations. We don't go into how this should or could affect the trips rather ask what this means about their predictions. This is a big dilemma of subject versus object simplicity. There is no doubt, the earth brother is simpler but does it mean that his prediction that is role as subject is true about the more complex brother, or the more complex brother's prediction will be true because it is about a simpler object.
The fifth level is again a simple fact like the third was. The subject simplicity overrides the object. The non accelerating brother was strictly a fix system so his predictions about the patchworked more complex trips of the other are still true. We have to see that either choice was a sacrifice, a contradiction. Now with this choice, the accelerations are not that important to destroy the earth brother's view, so they can be ignored simply because they were short, but they were important or crucial enough to totally invalidate the other's view.
The sixth level is that we look for explanations. Namely we want to see how the accelerations could not merely invalidate the returning brother's view about his brother but rather explain his brother's view. We realise the fundamental new conflict that the youngening is proportional to the fix speed trips that can be changed with same accelerations. So the only possible explanation is that accelerations affect the aging rate and then these remain in fix speed trips. This has a nice coincidence with what we already saw, namely that the last stop to stay on earth was immaterial. Thus the initial acceleration caused the youngening while going away and the returning acceleration caused the youngening while going home.
The seventh level is an attack against this simplistic explanation. We envision an experiment without first acceleration at all. The rocket merely passes by earth and a clock in it is zeroed to an earth clock. Then only return acceleration is needed. Amazingly same end result happens. The returning clock slows down, already in the going away period too. This means that the return as a decision or process affected the past, caused youngening under the earlier trip.
The eight level is a new explanation, that makes certain sense out of the mayhem we created previously. The slow down and reverse can be regarded as a fix force field, namely as an imaginary gravitation pulling the rocket backwards. General relativity claims that gravitation bends space time and the changes of times are related to energy potentials. Just as a ball thrown up must come down, similarly the trip before and after the imaginary gravitational field are symmetrical too.
The ninth level is the realization that we can create many seemingly contradictory explanations. There is no final explanation because we are trying to explain something that can not be explained. The fundamental fact that time distortion is accumulating can not be explained because it changes from a mere observational distortion into a seemingly more substantial reality. To catch this transition perfectly would mean a definite distinction between alternative descriptions. This of course contradicts relativity itself. So the Twin Paradox is merely the old accumulation paradox with a new emphasis. We should embrace it because it makes us conscious of the original.
Relativity is unfinished, our knowledge about time is unfinished.
The tenth level is the most important. It is the insanity of Formalism entering this paradox. The stupidity involved in the mentioned $E = m c^2$ debate is nothing

compared to this. The sick need to be on top of everything, to say that it all boils down to see it correctly, shows much more how these people obey the abstractions blindly. Blindly here means not just not connecting with their own intuitions, but also not connecting to the bigger pictures. What is this physics for? What are we doing in general? Most importantly though not connecting to the fundamental goal, to transfer understanding to elevate humans from slavery.

The big picture in science is the shift from the old naïve reality view. This was a belief that a sharp line exists between description and reality. The theories themselves suggested this view. A place is obviously described by different coordinates in different systems but distances and angles should be the same. Regarding moving systems and measuring distances and time intervals by signals from the endpoints, it's quite plausible that maybe intervals could be subjective too that is different in different systems. Local events, that is what happens to matter in a place at a time should still remain objective. Whether a star blows up to a supernova or not, should be objective though it may look different in different systems. After the early days of relativity, when these objectivity paradoxes were resolved, it seemed that only the Twin Paradox remained. This is a delusion! The very foundation that provides the invariances and the transformations, that is the fix speed of light is a paradox.

Just because we accept something or get used to it doesn't make it more plausible. The Formalist doesn't merely accept these abstractions, he wants to erase all problematicalness around it. But this tendency is hidden. The taste for abstractions is a certain plus. It's a speculativeness. But for Formalists it is a speculativeness with a dark twist. The same is present in sceptical or humorous people in general. The smartness hides the dark weakness. Though these people seem confrontational, they are obeying society. In fact they are one of the strongest tools of society to spread the illusion of openness and the unavoidability of "the ways things are". They think that stupid people in power is the source of all problems. They make fun of themselves too, which makes the deeper motivating force, the conservation of privileges even less apparent. So, the blindness to see that society is the real problem not people, may seem as a strange non speculativeness from such speculative people but it melts with raw selfishness.

Returning to the wider break down of the old objectivity principle, it can be seen in Quantum Mechanics and New Math too. In the Twin Paradox the confrontation with different age is the sharpest example of a fact, a reality, that we want to justify because it came out of nothing, through mere observational abstractions. Remember, that the $E_0 = m_0 c^2$ total energy came out the same way. It is a mysterious insight into matter by purely external abstractions. But here with time we obtained the opposite. It is not fix. So we want to "catch" how the aging as process alters by motion. Not only explanations fail to do this but if we try to follow communications between the brothers we don't get closer either. In a sense, we can say that such communications are restricted interactions and restrictions on interactions allow the separate flows of times. A totally restricted interaction, that provides for anyone time travel into the future even without a rocket is simply being frozen. We don't find this contradictory because an objective cause of the altered aging is present. The incommunicado seems a mere side effect. Maybe there is time travel into the past too. The fact that we don't know why this should be impossible or not, shows too that the Twin Paradox is unresolved.

So, will only the future tell the truth? Well if you read carefully this article you'll be not surprised that in my view, the truth is very much in the present.